Karlsruhe Institute of Technology

PaCHash: Packed and Compressed Hash Tables

First ACDA Workshop in Aussois

Florian Kurpicz (joint work with Hans-Peter Lehmann and Peter Sanders)

Motivation

- hash table for objects of **variable** size
- storing objects in external memory
- very small internal memory data structure

- retrieve objects in 1 I/O
- consecutive I/O (*)

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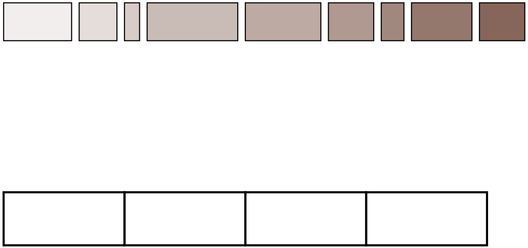
- objects of size 256 bytes
- in blocks of size 4096 bytes
- internal space for one block

	Method	l_b	α	I/Os
fixed	Larson et al. [LR85]	96	<96	1
	SILT SortedStore [Lim+11]	51	100	1
	Linear Separator [Lar88]	8	85	1
	Separator [GL88; LK84]	6	98	1
	Robin Hood [Cel88]	3	99	1.3
	Ramakrishna et al. [RT89]	4	80	1
	Jensen, Pagh [JP08]	0	80	1.25
	Cuckoo [Aza+94; Pag03]	0	<100	2
	PaCHash, $a = 1$	2	100	2*
	PaCHash, $a = 8$	5	100	1.13*
variable	SILT LogStore [Lim+11]	832	100	1
	SkimpyStash [DSL11]	32	≤ 98	8
	PaCHash, $a = 1$	2	99.95	2.06*
	PaCHash, $a = 8$	5	99.95	1.19*

PaCHash Overview



objects of variable size

EM

PaCHash Overview

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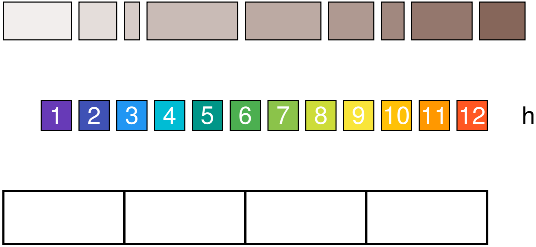

objects of variable size

hash function $h: K \rightarrow 1..am$

EM

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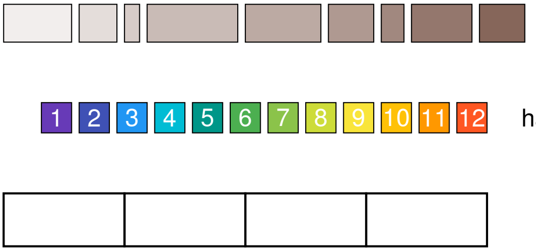


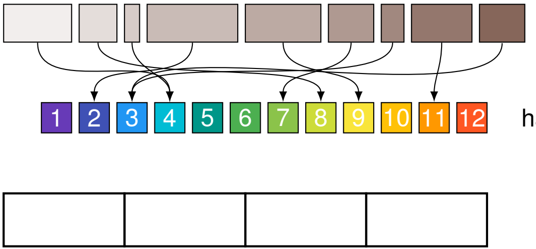
objects of variable size

tuning parameter

hash function $h\colon K \to 1..am$

EM

PaCHash Overview



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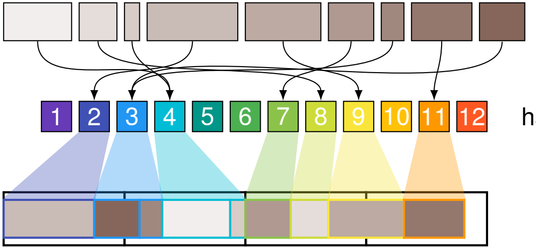

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sorted objects in EM

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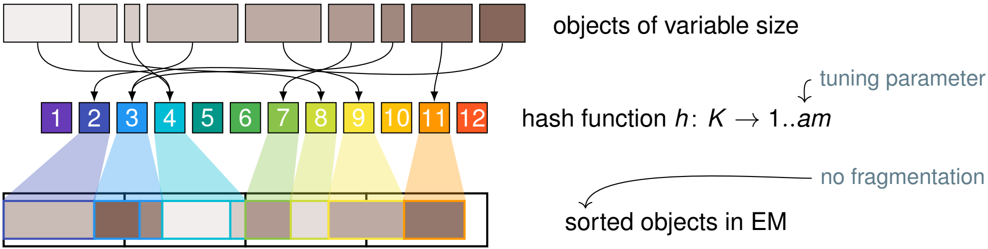
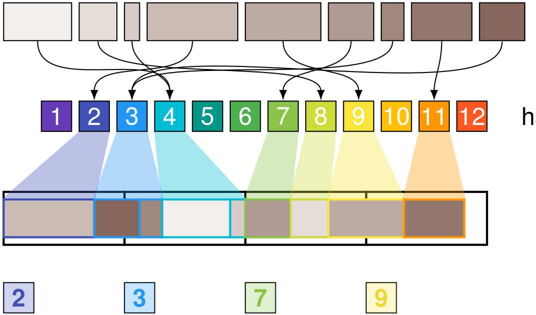


PaCHash Overview



objects of variable size

tuning parameter

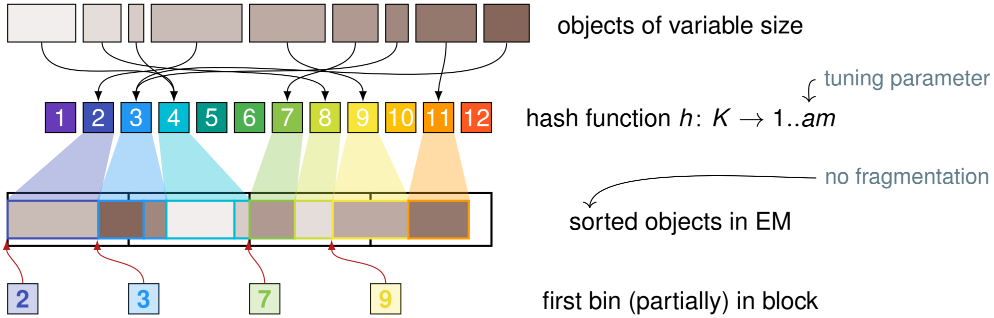
hash function $h\colon K \to 1..am$

no fragmentation

sorted objects in EM

first bin (partially) in block

PaCHash Overview



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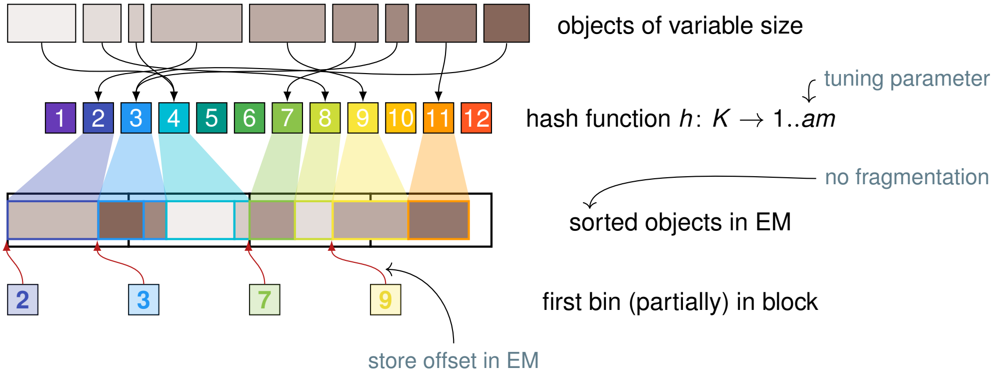

Setting

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- offset requiring $d = 0..\lceil \log \mathrm{B} \rceil$ bits
- $\bar{\mathrm{B}} = \mathrm{B} - d$ remaining per EM block
- n objects of total size N
- $m = N/\bar{\mathrm{B}}$ blocks in EM
- $p = \langle p_1, \ldots, p_m \rangle$ internal memory

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- external memory blocks of size $\mathtt{B}$

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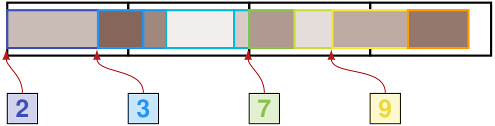
- alternatives for different object sizes

d	Case Description
0	Identical object sizes, zero terminated strings and analogous cases
$\lceil \log(w+1) \rceil$	Objects that use variable bit-length encoding with $\leq w \leq \mathtt{B}$ bits
$\lceil \log(W/w+1) \rceil$	Objects of size divisible by w with $W = \min(\mathtt{B}, \mathtt{max}$ object size)
$\lceil \log(\mathtt{B}) \rceil$	Explicit storage of a starting position of a bin

Finding Blocks

Query Algorithm



- $b = h(x)$
- find predecessor p_i of b
- if $p_i = b$ let $i = i - 1$
- find first j with $p_j > b$

Finding Blocks

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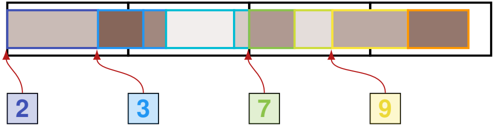

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Uncompressed Bit Vector

- finding predecessor in bit vector in $O(1)$ time
- requires $m(a+1) + o(m(a+1))$ bits of space (uncompressed)

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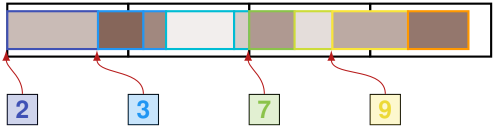

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Elias-Fano Coding

- using Elias-Fano coding saves space
  - given k monotonic increasing integers in $1..u$
  - store $\log k$ MSBs as bit vector
  - store $\log(u/k)$ LSBs plain
  - $k(2 + \log(u/k)) + 1 + o(k)$ bits in total
- predecessor in $O(\log(u/k))$ time
- finding range of ones in bit vector

Internal Memory Data Structure

Lemma: Space with Elias-Fano Coding

When using Elias-Fano coding [Eli74; Fan71] to store p, the index needs $2 + \log a + o(1)$ bits of internal memory per block.

Proof

- m blocks containing bins $1..am$
- requires $m(2 + \log a) + 1 + o(m)$ bits

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Lemma: Expected Predecessor Time

When using Elias-Fano coding to store p, the range of blocks containing the bin of an object x can be found in expected constant time.

Proof

- consider $\log m$ MSB
- let bin b_x have MSBs equal to u
- expected size $\mathbb{E}(Y_u)$ of all bins with MSB u that are $< b_x$ is

$$\sum_{y \in S} |y| \cdot \mathbb{P}(h(y) \text{ w/ MSB} = u; h(y) < h(x))$$

$$\leq \sum_{y \in S} |y| \cdot \mathbb{P}(h(y) \text{ w/ MSB} = u)$$

$$= \frac{1}{m} \sum_{y \in S} |y| = \frac{m\bar{B}}{m} = \bar{B}$$

Loading Blocks from External Memory

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Lemma: Additional Blocks Loaded

Retrieving an object x of size $|x|$ from a PaCHash data structure loads $\leq 1 + |x|/\overline{\mathrm{B}} + 1/a$ consecutive blocks from the external memory in expectation (setting $|x| = 0$ if x is not in the table).

Proof

- expected size of block $b_x = h(x)$

$$
\begin{aligned}
\mathbb{E}(|b_x|) &= |x| + \sum_{y \in S, y \neq x} |y| \mathbb{P}(y \in b_x) \\
&\leq |x| + \sum_{y \in S} |y| \mathbb{P}(y \in b_x) \\
&= |x| + \sum_{y \in S} |y| \cdot \frac{1}{am} = |x| + \frac{\overline{\mathrm{B}}}{a}
\end{aligned}
$$

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Proof (cnt.)

- expected number of blocks overlapped by bin

$$\begin{aligned}\mathbb{E}(X) &= 1 + (\mathbb{E}(|b_x|) - 1)/\overline{\text{B}} \\ &= 1 + |x|/\overline{\text{B}} + 1/a - 1/\overline{\text{B}}\end{aligned}$$

- $\mathbb{P}(\text{bin and block border align}) = 1/\overline{\text{B}}$

Experimental Evaluation

Hardware and Software

- Intel i7 11700 with base clock speed of 2.5 GHz
- 1 TB Samsung 980 Pro NVMe SSD
- Ubuntu 21.10 with Linux 5.13.0
- `io_uring` for I/O operations
- GCC 11.2.0 (`-O3 -march=native`)
- $\mathtt{B} = 4096$ bytes

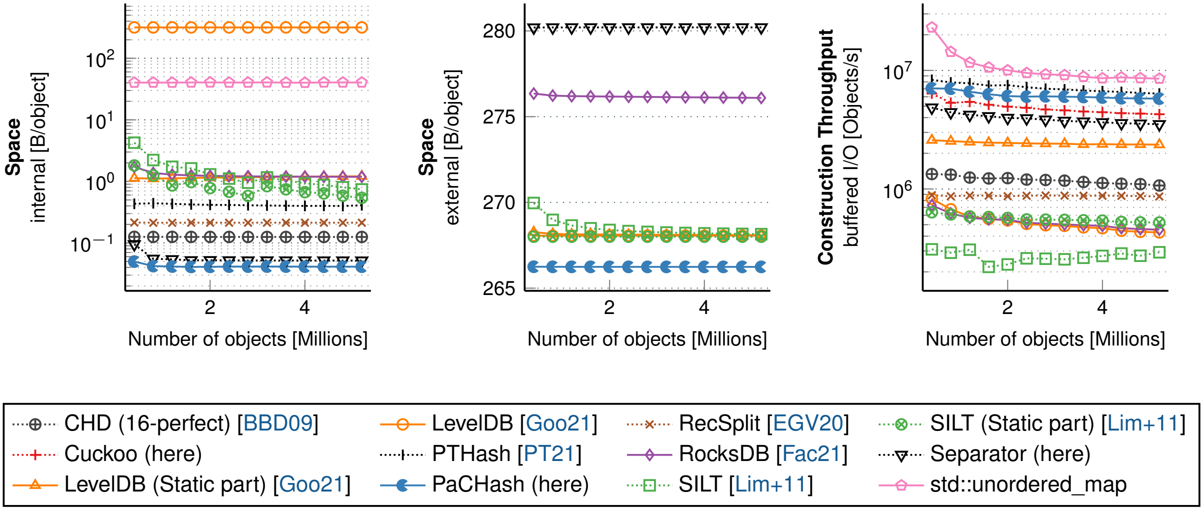
Objects

- here only fixed size
- more in the paper

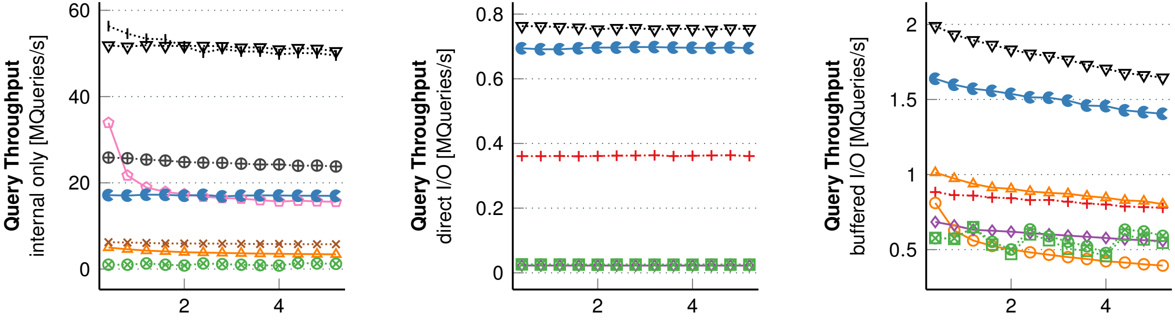
Competitors

- LevelDB [Goo21]
- RocksDB [Fac21]
- SILT [Lim+11].
- `std::unordered_map`
- RecSplit [EGV20]
- CHD [BBD09; CR+12]
- PTHash [PT21]

Construction

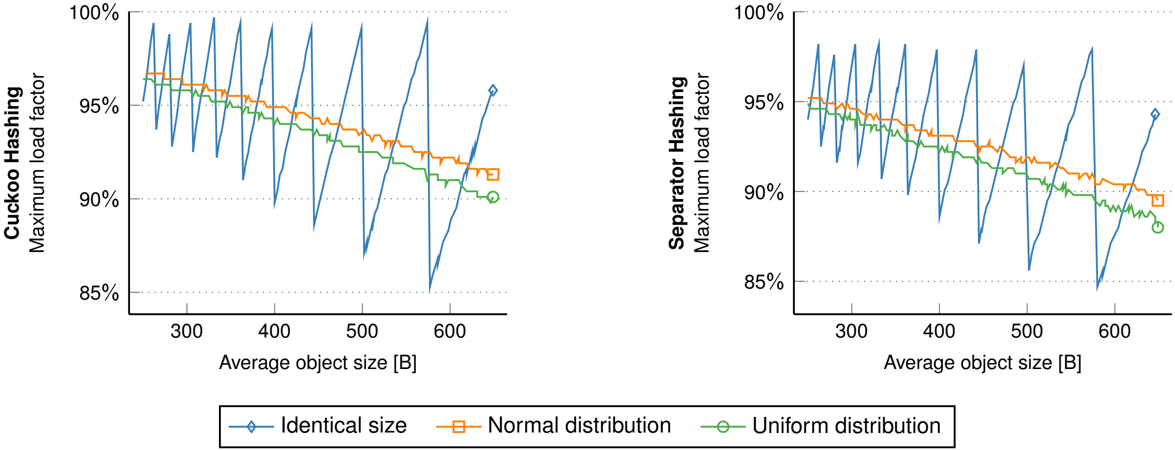


Queries



CHD (16-perfect) [BBD09]
Cuckoo (here)
LevelDB (Static part) [Goo21]
LevelDB [Goo21]
PTHash [PT21]
PaCHash (here)
RecSplit [EGV20]
RocksDB [Fac21]
SILT [Lim+11]
SILT (Static part) [Lim+11]
Separator (here)
std::unordered_map

Maximum Load Factor of Competitors



Alternative Internal Memory Data Structure

Lemma: Space with Succincter

When using Succincter [Pat08] to store p, the index needs $1.4427 + \log(a + 1) + o(1)$ bits of internal memory per block.

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Entropy Encoding

- encode positions directly
- compress bit vector using entropy coding
- currently very naive Huffman codes on blocks of size 8, 16, 32, or 64

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Structure of Bit Vector

- runs of `0`s
- runs of `10`s
- sometimes additional `1`s

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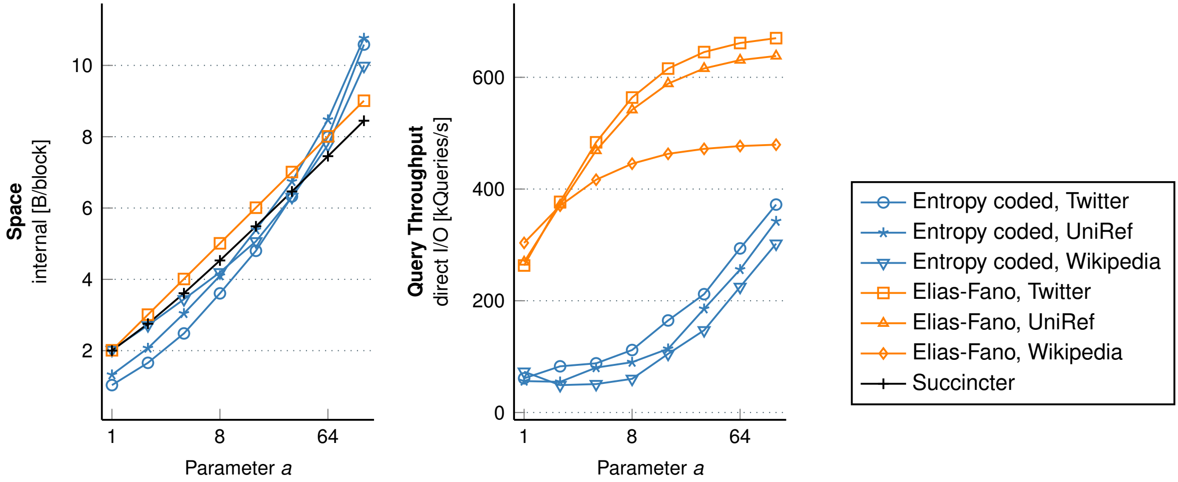
Open Question

- better compression using the structure
- better as in faster to decompress
- at most the size of the entropy encoding

Compressing Internal Memory Data Structure



Conclusion

- static hash table for objects of variable size
- constant number of bits per EM block
- outperforming competitors (variable size)
- matching and outperforming competitors (fixed size)

- code available under GPLv3 license
- `https://github.com/ByteHamster/PaCHash`
- preprint available in arxiv

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Bibliography I

[Aza+94] Yossi Azar, Andrei Z. Broder, Anna R. Karlin, and Eli Upfal. “Balanced allocations (extended abstract)”. In: *STOC*. ACM, 1994, pages 593–602. DOI: 10.1145/195058.195412.

[BBD09] Djamal Belazzougui, Fabiano C. Botelho, and Martin Dietzfelbinger. “Hash, Displace, and Compress”. In: *ESA*. Volume 5757. Lecture Notes in Computer Science. Springer, 2009, pages 682–693. DOI: 10.1007/978-3-642-04128-0_61.

[Cel88] Pedro Celia. *External Robin Hood Hashing*. Technical report. Computer Science Department, Indiana University. TR246, 1988.

[CR+12] Davi de Castro Reis, Djamel Belazzougui, Fabiano Cupertino Botelho, and Nivio Ziviani. *CMPH - C Minimal Perfect Hashing Library*. http://cmph.sourceforge.net/. 2012.

[DSL11] Biplob K. Debnath, Sudipta Sengupta, and Jin Li. “SkimpyStash: RAM space skimpy key-value store on flash-based storage”. In: *SIGMOD Conference*. ACM, 2011, pages 25–36. DOI: 10.1145/1989323.1989327.

Bibliography II

[EGV20] Emmanuel Esposito, Thomas Mueller Graf, and Sebastiano Vigna. "RecSplit: Minimal Perfect Hashing via Recursive Splitting". In: *ALENEX*. SIAM, 2020, pages 175–185. DOI: 10.1137/1.9781611976007.14.

[Eli74] Peter Elias. "Efficient Storage and Retrieval by Content and Address of Static Files". In: *J. ACM* 21.2 (1974), pages 246–260. DOI: 10.1145/321812.321820.

[Fac21] Facebook. *RocksDB. A Persistent Key-Value Store for Fast Storage Environments*. https://rocksdb.org. 2021.

[Fan71] Robert Mario Fano. *On the number of bits required to implement an associative memory*. Technical report. Project MAC, Memorandum 61". MIT, Computer Structures Group, 1971.

[GL88] Gaston H. Gonnet and Per-Åke Larson. "External hashing with limited internal storage". In: *J. ACM* 35.1 (1988), pages 161–184. DOI: 10.1145/42267.42274.

[Goo21] Google. *LevelDB is a Fast Key-Value Storage Library Written at Google*. https://github.com/google/leveldb. 2021.

Bibliography III

[JP08] Morten Skaarup Jensen and Rasmus Pagh. “Optimality in External Memory Hashing”. In: *Algorithmica* 52.3 (2008), pages 403–411. DOI: `10.1007/s00453-007-9155-x`.

[Lar88] Per-Åke Larson. “Linear Hashing with Separators - A Dynamic Hashing Scheme Achieving One-Access Retrieval”. In: *ACM Trans. Database Syst.* 13.3 (1988), pages 366–388. DOI: `10.1145/44498.44500`.

[Lim+11] Hyeontaek Lim, Bin Fan, David G. Andersen, and Michael Kaminsky. “SILT: a memory-efficient, high-performance key-value store”. In: *SOSP*. ACM, 2011, pages 1–13. DOI: `10.1145/2043556.2043558`.

[LK84] Per-Åke Larson and Ajay Kajla. “File Organization: Implementation of a Method Guaranteeing Retrieval in One Access”. In: *Commun. ACM* 27.7 (1984), pages 670–677. DOI: `10.1145/358105.358193`.

[LR85] Per-Åke Larson and M. V. Ramakrishna. “External Perfect Hashing”. In: *SIGMOD Conference*. ACM Press, 1985, pages 190–200. DOI: `10.1145/318898.318916`.

Bibliography IV

[Pag03] Rasmus Pagh. “Basic External Memory Data Structures”. In: *Algorithms for Memory Hierarchies*. Volume 2625. Lecture Notes in Computer Science. Springer, 2003, pages 14–35. DOI: 10.1007/3-540-36574-5_2.

[Pat08] Mihai Patrascu. “Succincter”. In: *FOCS*. IEEE Computer Society, 2008, pages 305–313. DOI: 10.1109/FOCS.2008.83.

[PT21] Giulio Ermanno Pibiri and Roberto Trani. “PTHash: Revisiting FCH Minimal Perfect Hashing”. In: *SIGIR*. ACM, 2021, pages 1339–1348. DOI: 10.1145/3404835.3462849.

[RT89] M. V. Ramakrishna and Walid R. Tout. “Dynamic External Hashing with Guaranteed Single Access Retrieval”. In: *FODO*. Volume 367. Lecture Notes in Computer Science. Springer, 1989, pages 187–201. DOI: 10.1007/3-540-51295-0_127.