

LIGHTWEIGHT DISTRIBUTED SUFFIX ARRAY CONSTRUCTION

Johannes Fischer *Florian Kurpicz*



0	1	2	3	4	5	6	7	8	9	10	11	12
T =	c	c	e	c	e	c	e	d	c	c	e	d \$

0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$
c	e	c	e	c	e	d	c	c	e	d	\$	
e	c	e	c	e	d	c	c	e	d	\$		
c	e	c	e	d	c	c	e	d	\$			
e	c	e	d	c	c	e	d	\$				
c	e	d	c	c	e	d	\$					
e	d	c	c	e	d	\$						
d	c	c	e	d	\$							
c	c	e	d	\$								
c	e	d	\$									
e	d	\$										
d	\$											
\$												

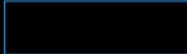
0 1 2 3 4 5 6 7 8 9 10 11 12

T =	c	c	e	c	e	c	e	d	c	c	e	d	\$
-----	---	---	---	---	---	---	---	---	---	---	---	---	----

SA =	12	0	8	1	3	9	5	11	7	2	4	10	6
------	----	---	---	---	---	---	---	----	---	---	---	----	---

\$	c	c	c	c	c	c	d	d	e	e	e	e	e
c	c	c	e	e	e	e	\$	c	c	c	d	d	d
e	e	e	c	c	d	d		c	e	e	\$	c	c
c	d	e	e	\$	c		e	c	d		c	c	c
e	\$	c	d		c		d	e	c		e	e	e
c	e	c	c		e		\$	d	c		d	d	\$
e	d	c	c		d			c	e				
d	c	e			\$			c	d				
c	c	d						e					\$
c	e				\$			d					
e	d							\$					
d					\$								
\$													

BULK SYNCHRONOUS PARALLEL MODEL

PE 1 

PE 2 

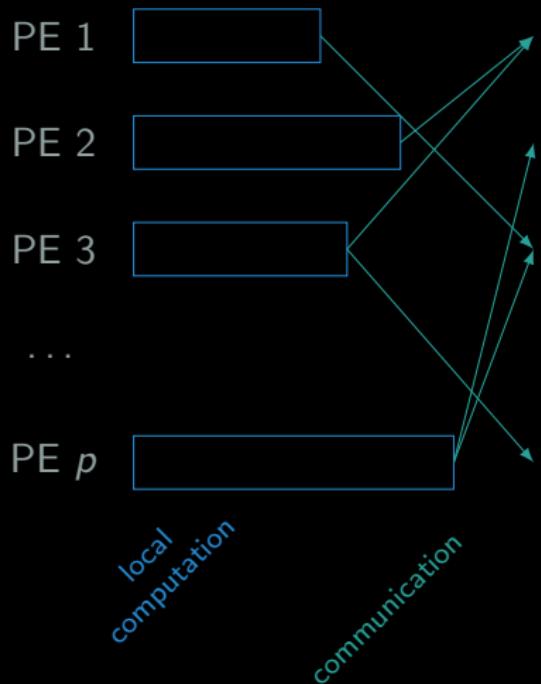
PE 3 

...

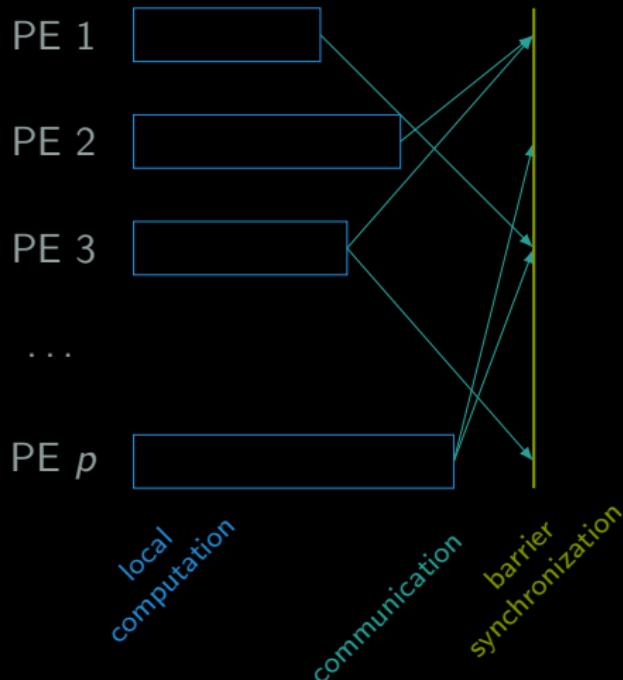
PE p 

local
computation

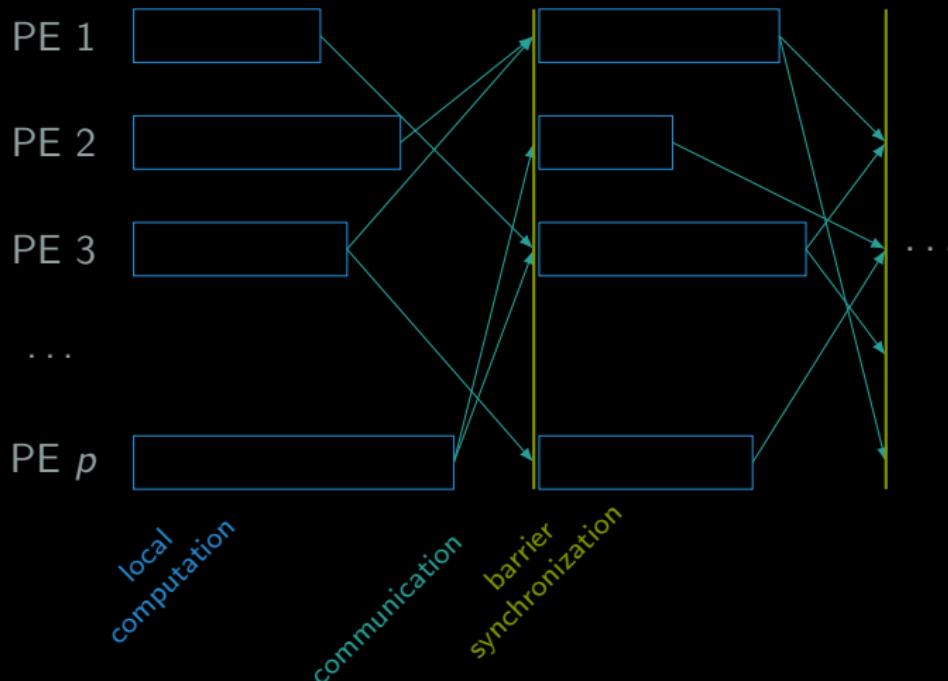
BULK SYNCHRONOUS PARALLEL MODEL

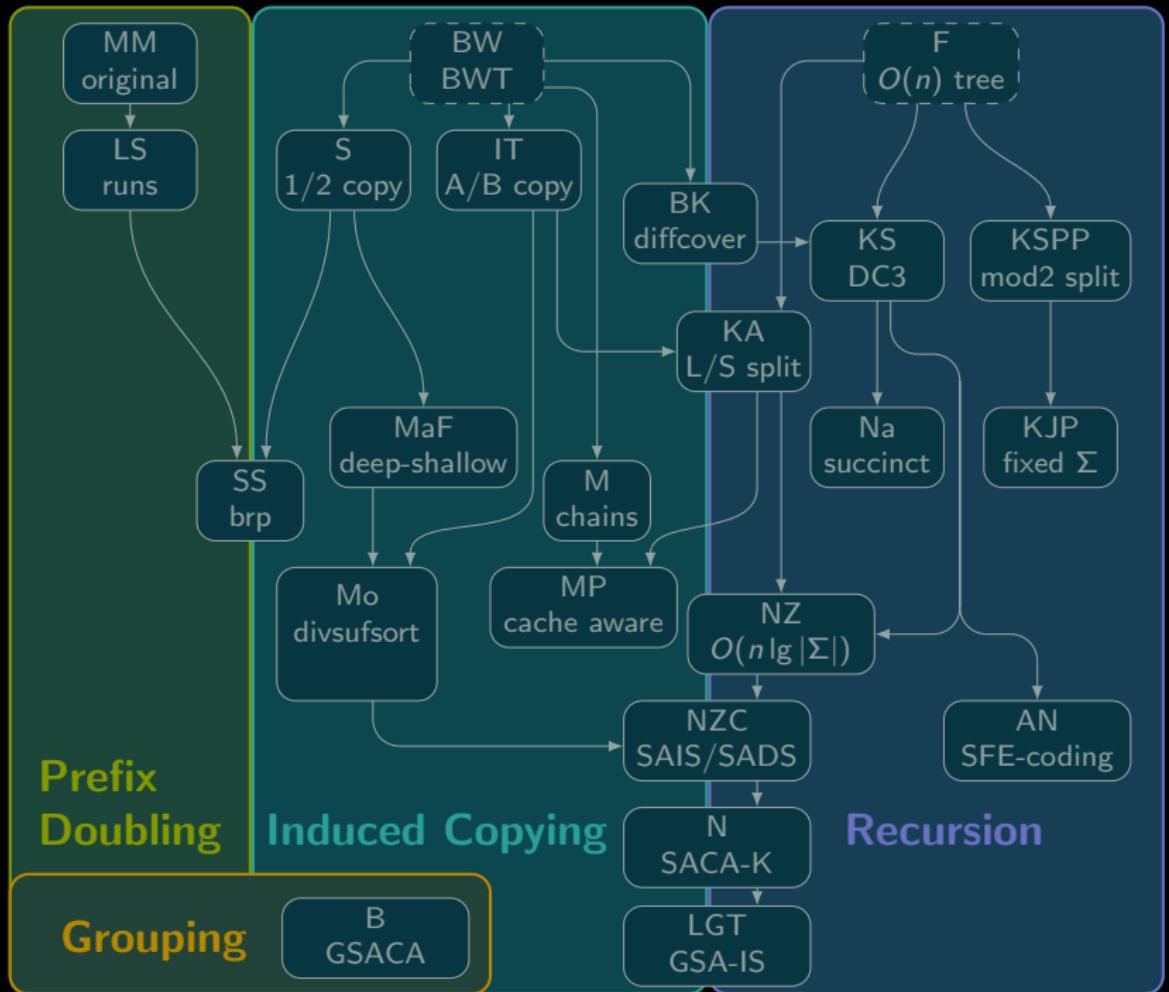


BULK SYNCHRONOUS PARALLEL MODEL



BULK SYNCHRONOUS PARALLEL MODEL





Prefix
Doubling

Induced Copying

Recursion

Grouping

B
GSACA

Prefix Doubling



Induced Copying



Recursion

[PST07]
[BFO13]
updated
1999

2009

2011

2016

Prefix Doubling



Induced Copying

this talk
divsufsort

Recursion



2009

2011

2016

[PST07]
[BFO13]
updated
1999

What is Induced Copying?

0 1 2 3 4 5 6 7 8 9 10 11 12

c	c	e	c	e	c	e	d	c	c	e	d	\$
---	---	---	---	---	---	---	---	---	---	---	---	----

12	0	8	1	3	9	5	11	7	2	4	10	6
----	---	---	---	---	---	---	----	---	---	---	----	---

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	e	\$	c	c	c	d	d
e	e	c	c	d	d	d	c	e	e	\$	c	c
c	d	e	e	\$	c	c	e	c	d	d	c	c
e	\$	c	d	c	c	c	d	e	c	c	e	e
c	e	c	c	e	e	e	\$	d	d	c	d	d
e	d	c	c	d	d	d	c	c	e	e	\$	\$
d	c	e	\$	c	c	c	c	d	d	\$		
c	c	d					e					
c	e	\$					d					
e	d						\$					
d	\$											
\$												

0 1 2 3 4 5 6 7 8 9 10 11 12

T =

c	c	e	c	e	c	e	d	c	c	e	d	\$
---	---	---	---	---	---	---	---	---	---	---	---	----

SA =

12	0	8	1	3	9	5	11	7	2	4	10	6
----	---	---	---	---	---	---	----	---	---	---	----	---

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	e	\$	c	c	c	d	d
e	e	c	c	c	d	d	c	e	e	e	\$	c
c	d	e	e	e	\$	c	c	e	c	d	c	c
e	\$	c	d	c	c	c	d	e	c	c	e	e
c	e	c	c	e	c	e	\$	d	c	c	d	\$
e	d	c	c	d	c	d	c	c	e	e	\$	
d	c	e	\$	c	e	\$	c	c	d	\$		
c	c	d		c	d		e					
c	e	\$		e	\$		d					
e	d			d								
d	\$											

0 1 2 3 4 5 6 7 8 9 10 11 12

T =

c	c	e	c	e	c	e	d	c	c	e	d	\$
---	---	---	---	---	---	---	---	---	---	---	---	----

SA =

12	0	8	1	3	9	5	11	7	2	4	10	6
----	---	---	---	---	---	---	----	---	---	---	----	---

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	c	e	e	e	e	\$	c	c	c	d	d
e	e	e	c	c	c	d	d	c	e	e	\$	c
c	c	d	c	c	d	d	c	c	e	c	d	c
e	e	d	e	e	e	\$	c	c	d	c	e	e
c	c	\$	c	d	c	c	c	d	e	c	d	\$
e	c		e	c	c	e	c	d	e	c	d	
d	e		d	c	d	d	\$	c	c	e	\$	
c	d		c	e	d	\$		c	d	d	\$	
c	d		c	d	\$							
e	d											
d	\$											

0 1 2 3 4 5 6 7 8 9 10 11 12

T =

c	c	e	c	e	c	e	d	c	c	e	d	\$
---	---	---	---	---	---	---	---	---	---	---	---	----

SA =

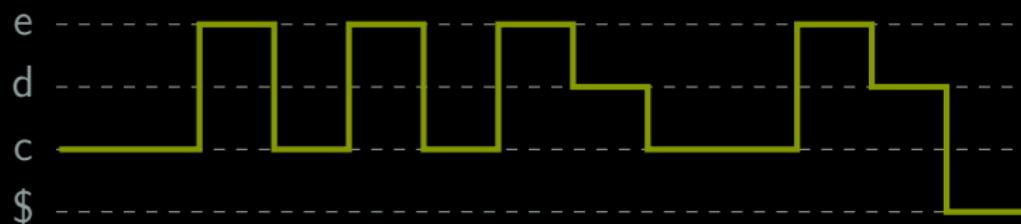
12	0	8	1	3	9	5	11	7	2	4	10	6
----	---	---	---	---	---	---	----	---	---	---	----	---

\$ c c c e c d \$ c c c c c c c
c e e e d \$ c e e e d \$ c c c
e c c c d \$ c d c c d e c c
c e e e d \$ c d c c d e c c
c e e e d \$ c d c c d e c c
d c c c d \$ c e c c d e c c
c c c d \$ c d c c d e c c
c e d \$ c d c c d e c c
d \$

Identify suffixes that must be sorted

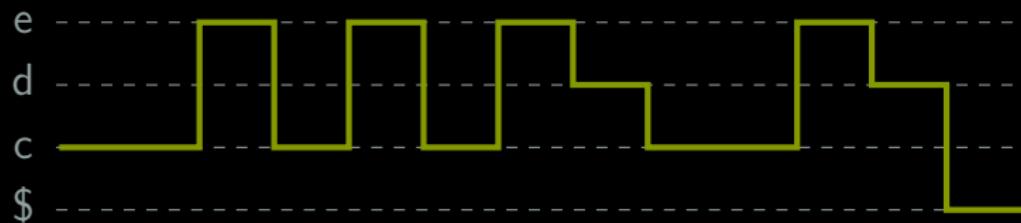
0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

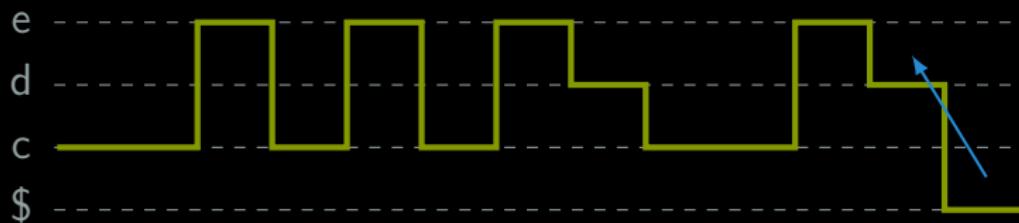
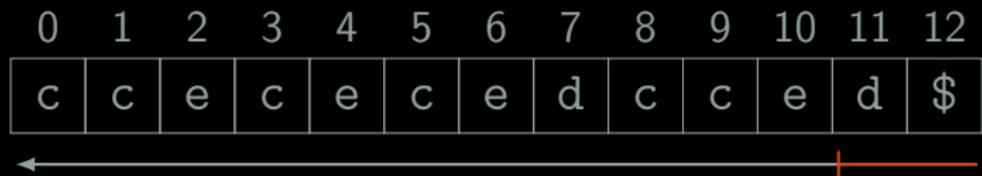
←

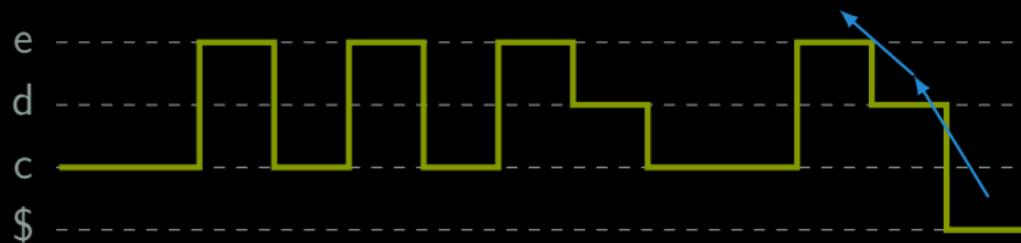
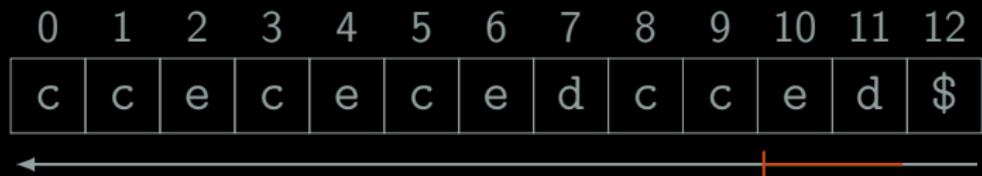


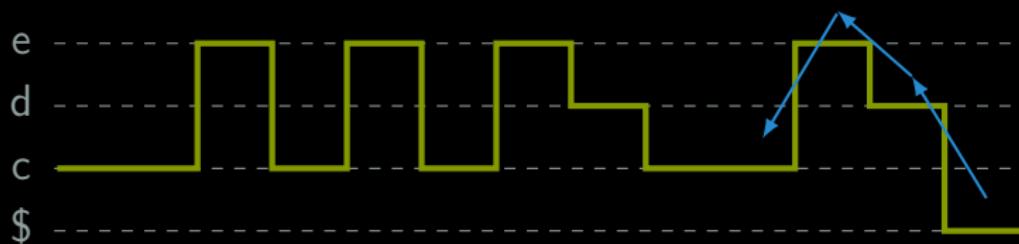
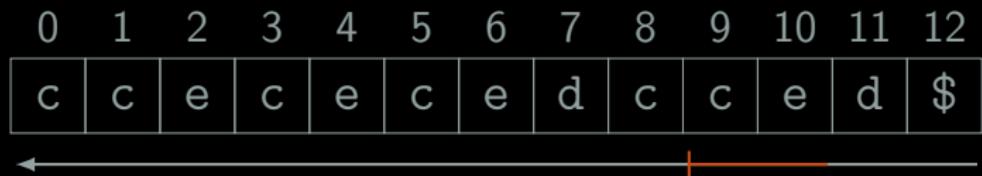
0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

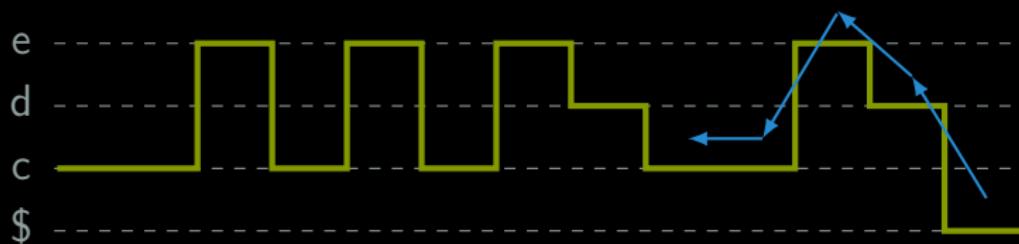
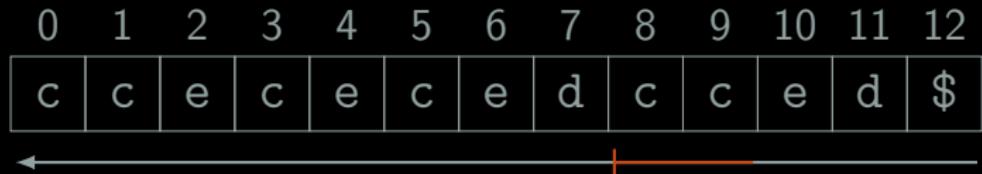
← + →





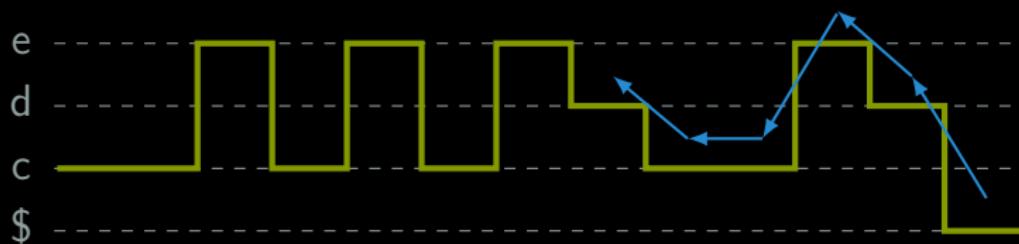




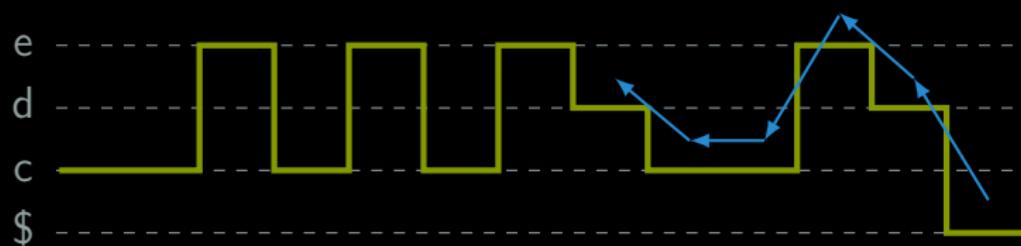


0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$



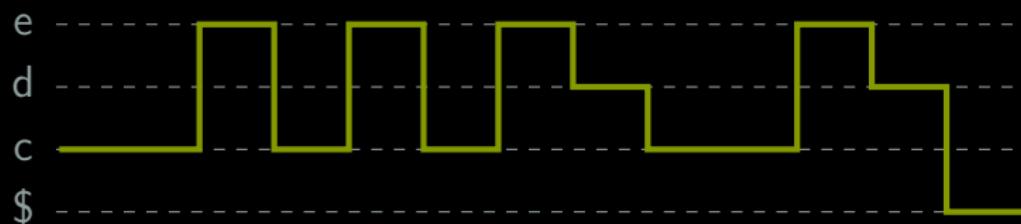


0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$



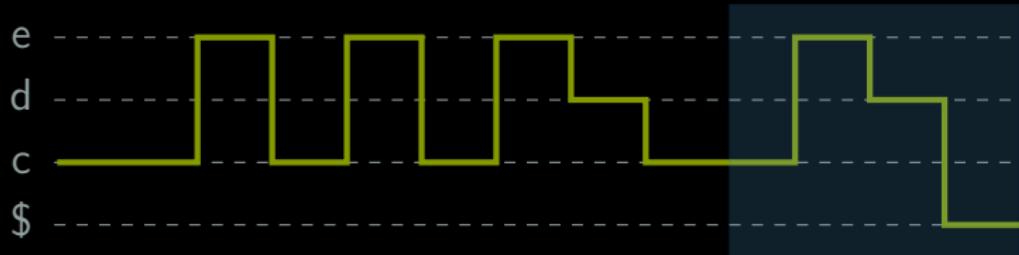
0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

←



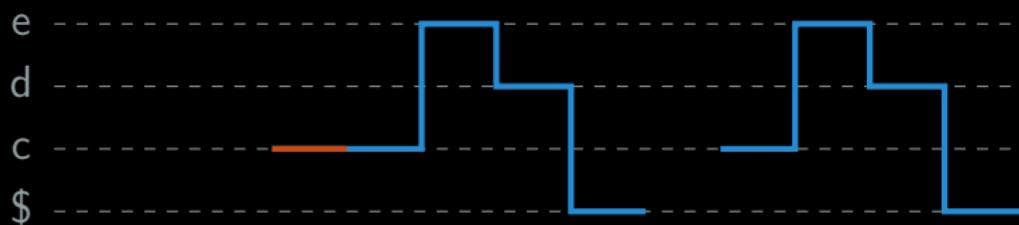
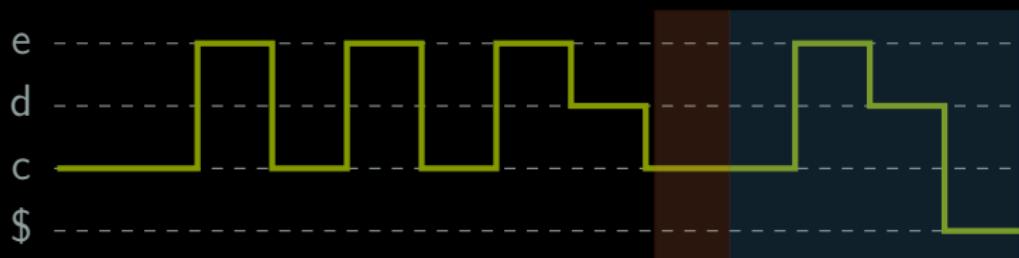
0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

←



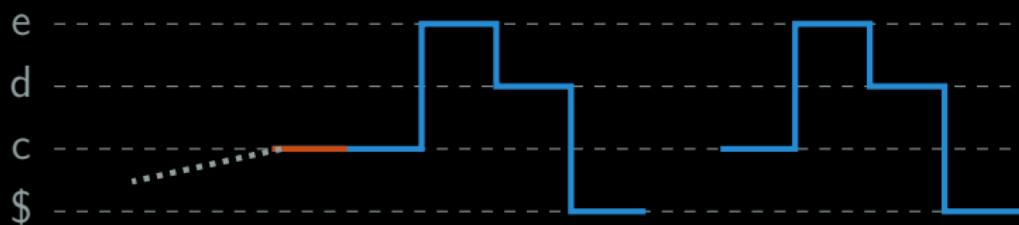
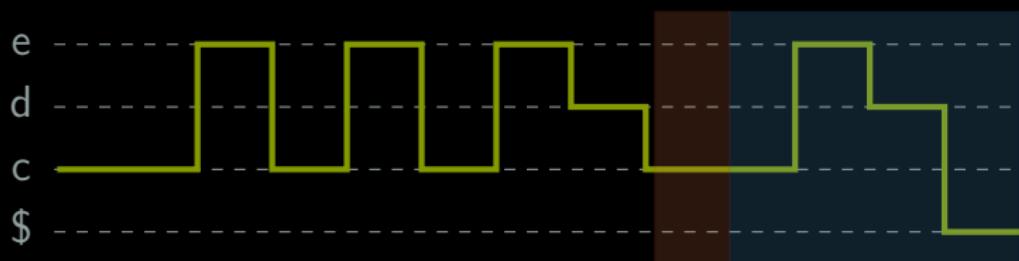
0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

←



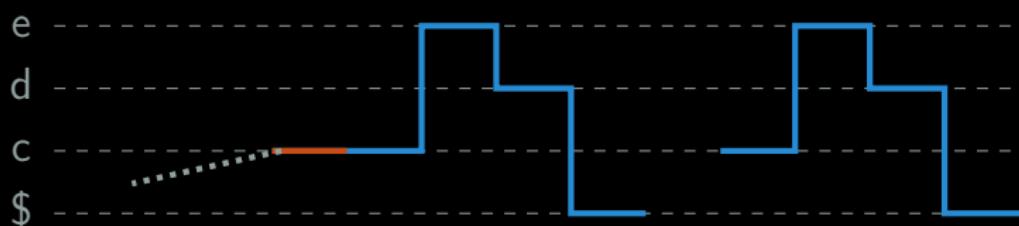
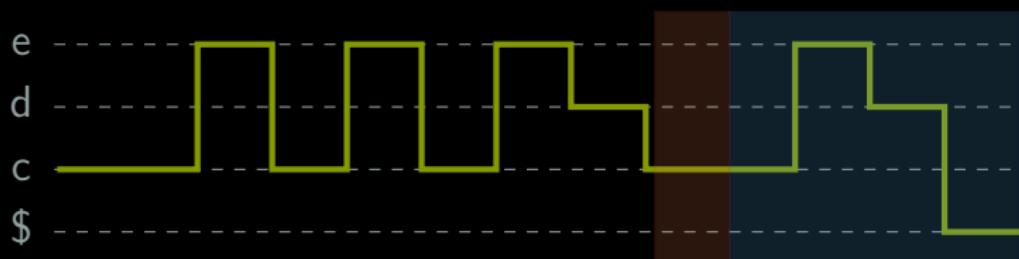
0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

←



0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

←



We only need to sort $\leq n/2$ suffixes

0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

↓

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|.....

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|.....

0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$



0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$



→ Distributed String Sorting

Distributed String Sample/Merge Sort

- ▶ Sort strings locally
- ▶ Compute splitters
- ▶ Distribute strings accordingly
- ▶ Merge received strings

Distributed String Sample/Merge Sort

- ▶ Sort strings locally → use string sorters by Bingmann and Rantala
- ▶ Compute splitters
- ▶ Distribute strings accordingly
- ▶ Merge received strings

Experiments

node two Intel Xeon E5-2640v4 (10 cores each) and 64 GB RAM

PE one MPI thread per CPU core

input real world texts (90 MB or 28 MB per PE) up to 115 GB

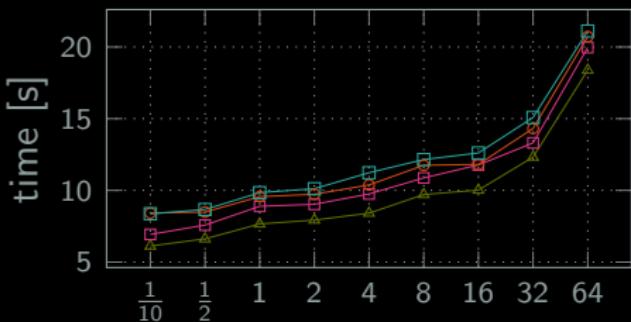
CC $\sigma = 242$

DNA $\sigma = 4$

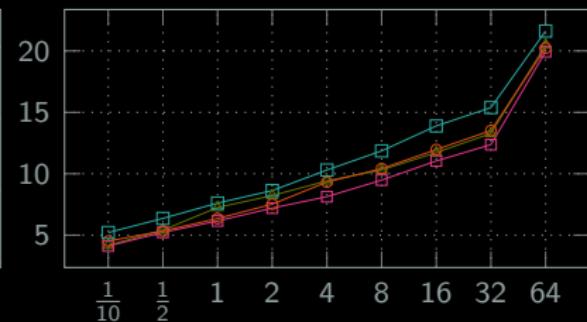
Prot $\sigma = 26$

Wiki $\sigma = 213$

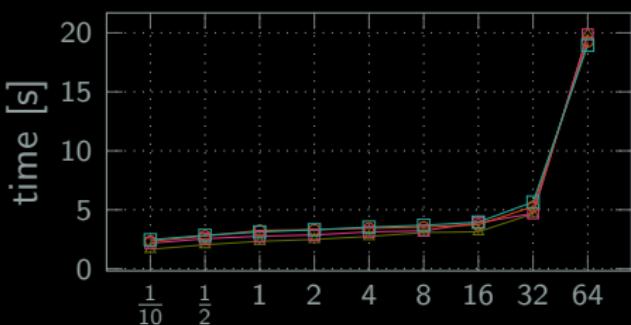
CC (\downarrow -substr.)



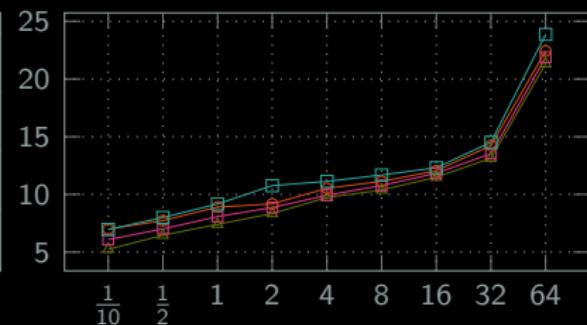
DNA (\downarrow -substr.)



PROT (\downarrow -substr.)



Wiki (\downarrow -substr.)



PEs p [$20 \cdot p$]

PEs p [$20 \cdot p$]

—▲— MSD radix sort —□— burstsort —■— multi-key —○— sample sort

0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$



0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$



0	0	2	1
---	---	---	---

0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$

↓

↓

↓

↓

|.....

|.....

|.....

|.....

0	0	2	1
---	---	---	---

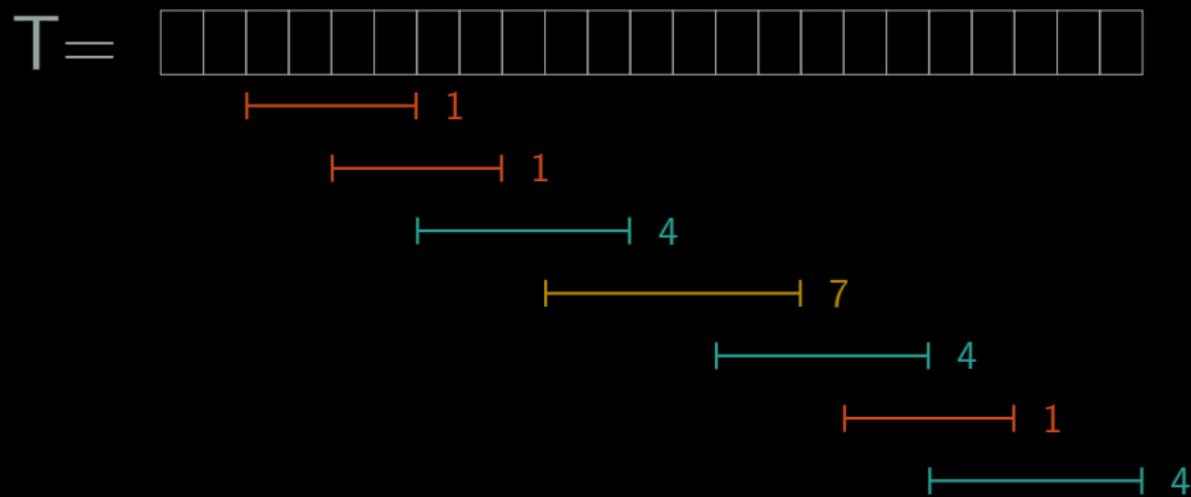
0	1	2	3	4	5	6	7	8	9	10	11	12
c	c	e	c	e	c	e	d	c	c	e	d	\$



0	0	2	1
---	---	---	---

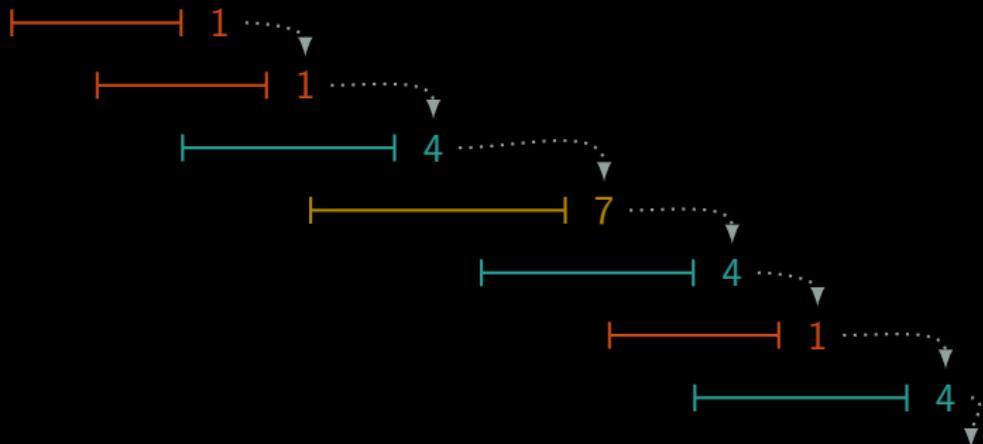
→ Distributed Suffix Array Construction

PREFIX DOUBLING

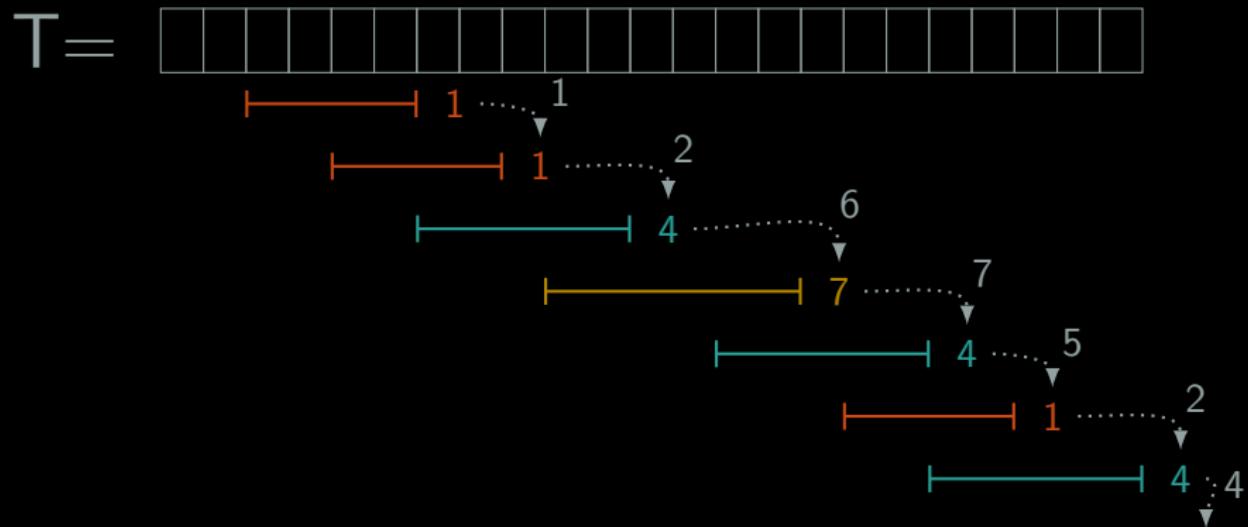


PREFIX DOUBLING

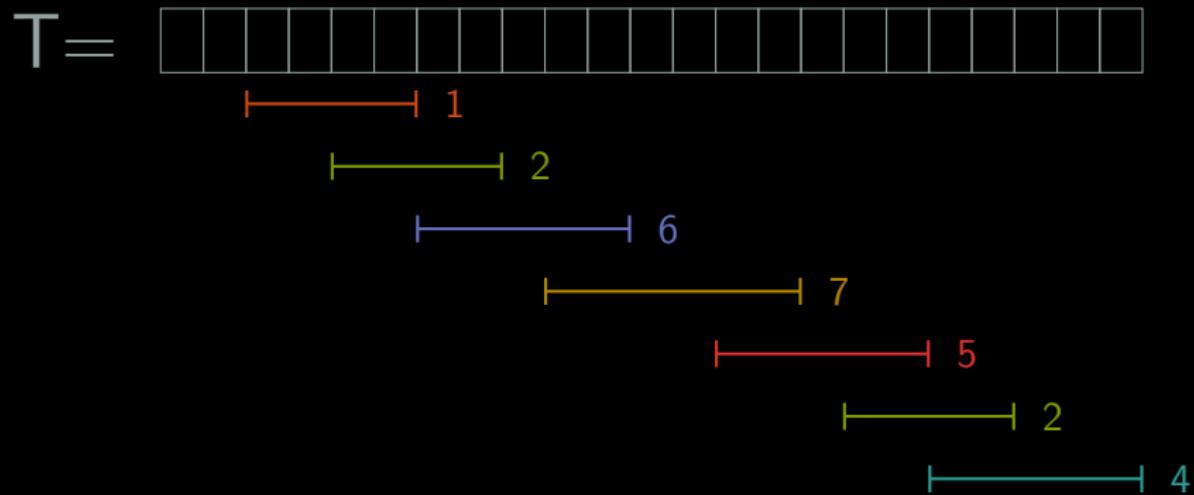
T = []



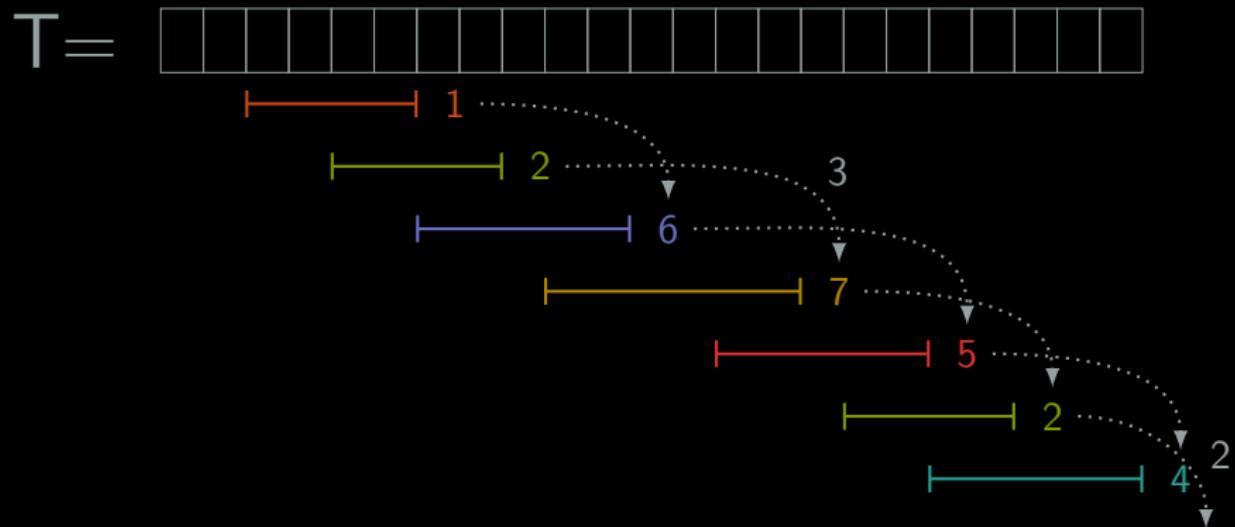
PREFIX DOUBLING



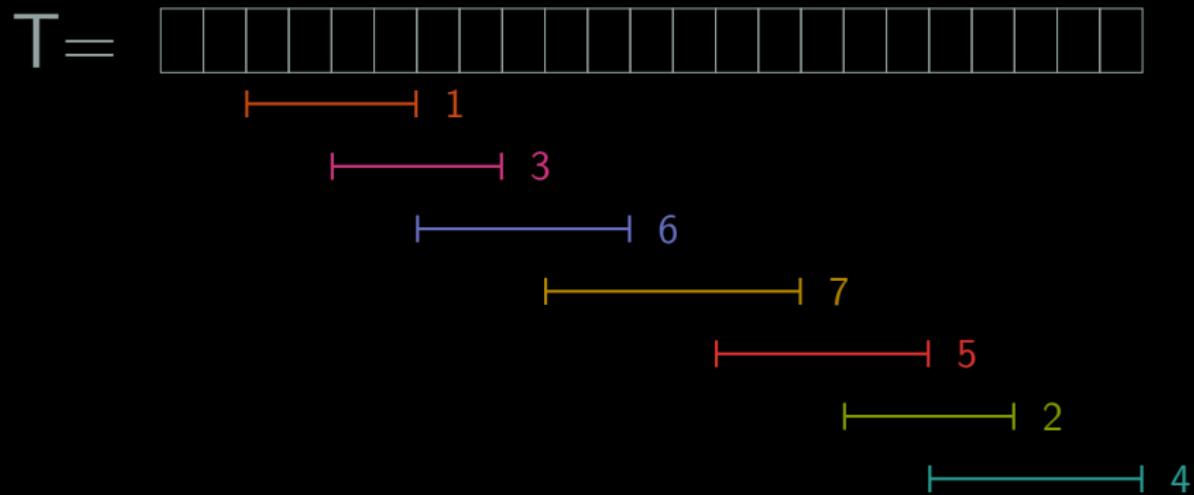
PREFIX DOUBLING



PREFIX DOUBLING



PREFIX DOUBLING



Induce other suffixes

Inducing \downarrow and \leftarrow

↔

0	1	2	3	4	5	6	7	8	9	10	11	12	
T =	c	c	e	c	e	c	e	d	c	c	e	d	\$
SA =	12	0	8	1	3	9	5	11	7	2	4	10	6

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	e	\$	c	c	c	d	d
e	e	c	c	d	d		c	e	e	\$	c	
c	d	e	e	\$	c		e	c	d		c	
e	\$	c	d		c		d	e	c		e	
c		e	c		e		\$	d	c		d	
e		d	c		d			c	e		\$	
d		c	e		\$			c	d			
c		c	d					e		\$		
c		e	\$					d				
e		d							\$			
d		\$										
\$												

Inducing \downarrow and \leftarrow

↔

0	1	2	3	4	5	6	7	8	9	10	11	12	
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d	\$
SA =	12	0	8	1	3	9	5	11	7	2	4	10	6

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	\$	c	c	c	d	d	d
e	e	c	c	d	d		c	e	e	\$	c	c
c	d	e	e	\$	c		e	c	d		c	c
e	\$	c	d		c		d	e	c		e	e
c		e	c		e		\$	d	c		d	d
e		d	c		d			c	e			\$
d		c	e		\$			c	d			
c		c	d					e				\$
c		e	\$					d				
e		d										\$
d		\$										
\$												

Inducing \downarrow and \leftarrow

↔

	0	1	2	3	4	5	6	7	8	9	10	11	12
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d	\$
SA =	12	0	8	1	3	9	5	11	7	2	4	10	6
\$	c	c	c	c	c	c	d	d	e	e	e	e	e
c	c	e	e	e	e	e	\$	c	c	c	d	d	d
e	e	c	c	c	d	d		c	e	e	\$	c	c
c	d	e	e	e	\$	c		e	c	d		c	c
e	\$	c	d		c			d	e	c		e	e
c	e	c			e			\$	d	c		d	d
e	d	c	c		d				c	e		\$	
d	c	e			\$				c	d			
c	c	d							e		\$		
c	e	\$							d				
e	d										\$		
d		\$											
\$													

Inducing \downarrow and \leftarrow

↔

0	1	2	3	4	5	6	7	8	9	10	11	12	
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d \$	
SA =	12	0	8	1	3	9	5	11	7	2	4	10	6

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	\$	c	c	c	d	d	d
e	e	c	c	c	d	d	c	e	e	\$	c	c
c	d	e	e	e	\$	c	e	c	d	c	c	c
e	\$	c	d	c	c	d	e	c	d	c	e	e
c	e	c	c	e		\$	d	c	d	c	d	d
e	d	c	c	d			c	e	c	e	\$	\$
d	c	e			\$			c	d			
c	c	d						e				\$
c	e	\$						d				
e	d											
d		\$										
\$												

Inducing \downarrow and \leftarrow

↔

	0	1	2	3	4	5	6	7	8	9	10	11	12
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d	\$
SA =	12	0	8	1	3	9	5	11	7	2	4	10	6
\$	c	c	c	c	c	c	c	d	d	e	e	e	e
c	c	c	e	e	e	e	e	\$	c	c	c	d	d
e	e	e	c	c	c	d	d	c	e	e	\$	c	c
c	c	d	c	c	d	d	c	c	e	c	d	c	c
e	e	\$	e	e	e	\$	c	c	d	e	c	e	e
c	c	c	c	c	c	c	e	c	\$	d	c	d	d
e	e	d	d	c	c	d	c	d	c	c	e	\$	c
d	d	c	c	c	c	\$	c	c	c	d	c	d	\$
c	c	c	c	c	c	c	e	d	e	d	\$	c	\$
c	c	d	d	d	d	d	d	\$	d	d	\$	c	\$
e	e	\$	\$	\$	\$	\$	\$	\$	\$	\$	\$	\$	\$
d	d	\$	\$	\$	\$	\$	\$	\$	\$	\$	\$	\$	\$

Inducing \downarrow and \leftarrow

↔

	0	1	2	3	4	5	6	7	8	9	10	11	12
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d	\$
SA =	12	0	8	1	3	9	5	11	7	2	4	10	6
\$	c	c	c	c	c	c	d	d	e	e	e	e	e
c	c	c	e	e	e	e	\$	c	c	c	d	d	d
e	e	e	c	c	d	d	c	e	e	\$	c	c	c
c	c	d	c	d	d	d	c	e	c	d	c	c	c
e	e	\$	c	e	e	c	c	d	e	c	e	e	e
c	c	c	c	c	e	e	\$	d	d	c	d	d	\$
e	e	d	d	c	c	d	\$	c	c	e	d	\$	
d	d	c	c	c	e	c	\$	c	c	d	\$		
c	c	c	c	d	d	\$				e			
c	c	e	d							d			
e	d	d	\$										

Inducing \downarrow and \leftarrow

↔

0	1	2	3	4	5	6	7	8	9	10	11	12	
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d \$	
SA =	12	0	8	1	3	9	5	11	7	2	4	10	6

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	\$	c	c	c	d	d	d
e	e	c	c	d	d		c	e	e	\$	c	c
c	d	e	e	\$	c		e	c	d		c	c
e	\$	c	d		c		d	e	c		e	e
c		e	c		e		\$	d	c		d	d
e		d	c		d			c	e		\$	
d		c	e		\$			c	d			
c		c	d					e		\$		
c		e	\$					d				
e		d							\$			
d		\$										
\$												

Inducing \downarrow and \leftarrow

↔

0	1	2	3	4	5	6	7	8	9	10	11	12
T =	c [↓]	c [↓]	e	c [↓]	e	c [↓]	e	d	c [↓]	c [↓]	e	d \$
SA =	12	0	8	1	3	9	5	11	7	2	4	10 6

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	e	\$	c	c	c	d	d
e	e	c	c	d	d		c	e	e	\$	c	
c	d	e	e	\$	c		e	c	d		c	
e	\$	c	d		c		d	e	c		e	
c		e	c		e		\$	d	c		d	
e		d	c		d			c	e		\$	
d		c	e		\$			c	d			
c		c	d					e			\$	
c		e	\$					d				
e		d										\$
d		\$										
\$												

Inducing \downarrow and \leftarrow

↔

0	1	2	3	4	5	6	7	8	9	10	11	12
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d \$
SA =	12	0	8	1	3	9	5	11	7	2	4	10 6

\$	c	c	c	c	c	c	d	d	e	e	e	e
c	c	e	e	e	e	e	\$	c	c	c	d	d
e	e	c	c	c	d	d		c	e	e	\$	c
c	d	e	e	e	\$	c		e	c	d		c
e	\$	c	d		c			d	e	c		e
c	e	c	c	e				\$	d	c		d
e	d	c	c	d	c	d			c	e		\$
d	c	e	d	\$					c	d		
c	c	d							e			\$
c	e	\$							d			
e	d											
d	\$											

Inducing \downarrow and \leftarrow

↔

0	1	2	3	4	5	6	7	8	9	10	11	12
T =	c \downarrow	c \downarrow	e	c \downarrow	e	c \downarrow	e	d	c \downarrow	c \downarrow	e	d \$
SA =	12	0	8	1	3	9	5	11	7	2	4	10 6

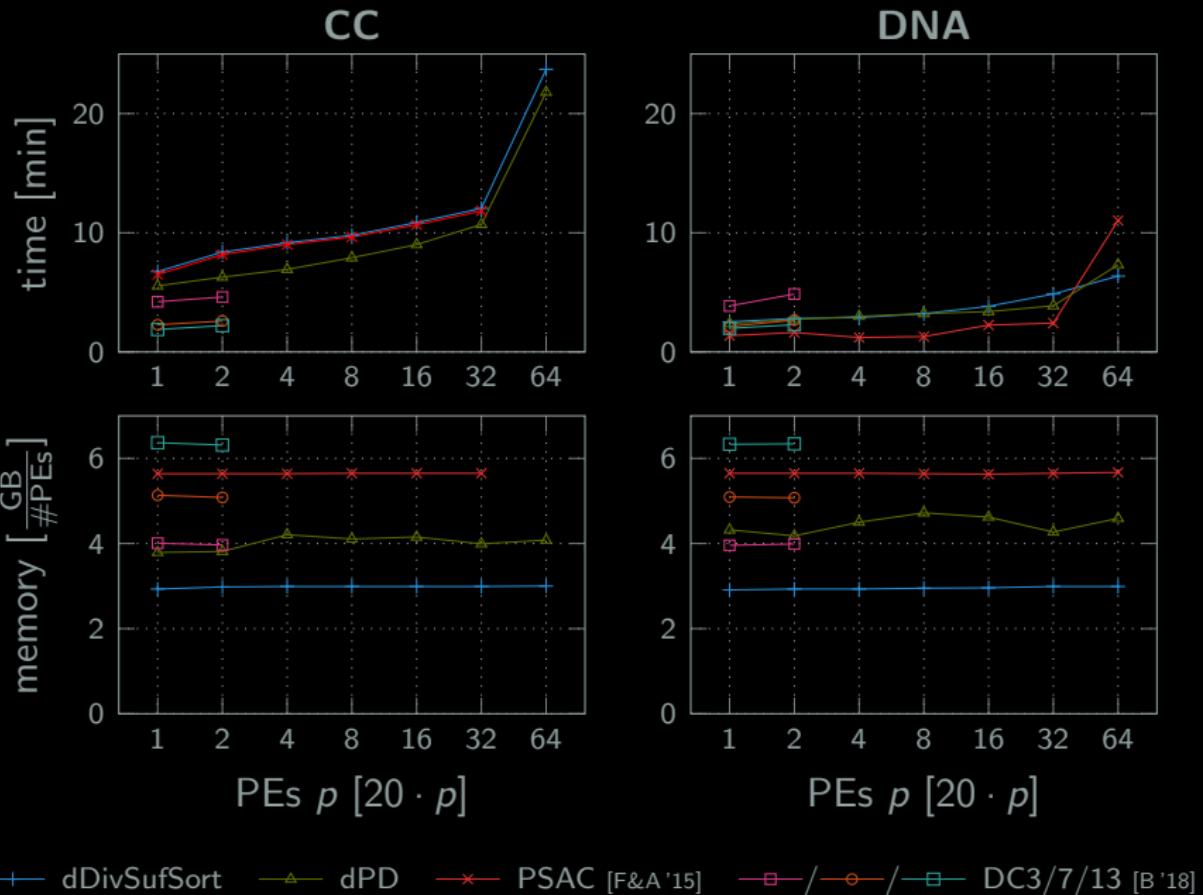
\$	c	c	c	c	c	d	d	e	e	e	e	e
c	c	c	e	e	e	\$	c	c	c	d	d	d
e	e	c	e	e	d	d	c	e	e	\$	c	c
c	d	c	c	d	d	c	e	c	d	c	c	c
e	\$	e	e	e	\$	c	d	e	c	e	e	e
c	c	c	c	c	c	c	c	\$	d	c	d	d
e	d	c	d	c	d	c	e	c	c	e	d	\$
d	\$	c	c	c	\$	c	d	c	c	d	\$	
c	c	c	d	c	c	c	e	d	c	e	d	\$
e	d	c	e	\$	d	\$	c	d	c	\$		
d	\$	d	d	\$								

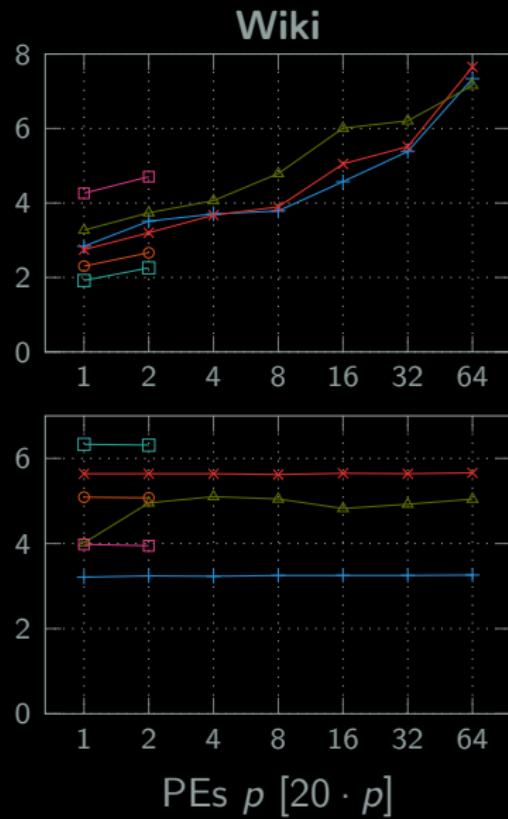
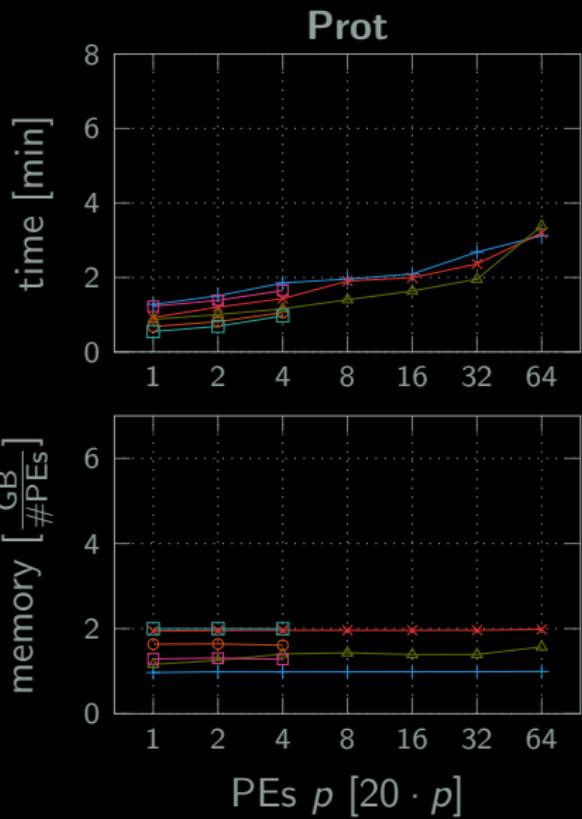
Key characteristics

- ▶ $2\sigma^2$ synchronizations
- ▶ work roughly equal on every MPI thread
- ▶ repetitions aaaa...aa require special case

Experiments

- ▶ same set-up as before
- ▶ measure construction time and
- ▶ memory peak





—+— dDivSufSort —△— dPD —×— PSAC [F&A '15] —□— / —○— / —□— DC3/7/13 [B '18]

Conclusion

- ▶ very memory efficient and
- ▶ reasonable fast distributed suffix array construction algorithms

Future Work

- ▶ computing the LCP-array

Conclusion

- ▶ very memory efficient and
- ▶ reasonable fast distributed suffix array construction algorithms

Future Work

- ▶ computing the LCP-array

Thank You