On the Benefit of Merging Suffix Array Intervals for Parallel Pattern Matching

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71. Workshop über Algorithmen und Komplexität

Notations

- Σ is the alphabet with $|\Sigma| = \sigma$
- $\$ \notin \Sigma$ and $\forall \alpha \in \Sigma : \$ <_{\text{lex}} \alpha$
- $T \in \Sigma^\star \cup \{\$\}$ and $P \in \Sigma^\star$
- $|T| = n$ and $|P| = m$
- p is the number of processors

Pattern Matching

Given a text T of length n and a pattern P of length m, find all occurrences of P in T.

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$P_1 = \texttt{b}$ and $P_2 = \texttt{a}$

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Sequential Times

Type	Query Time	Idea
exact	$\mathcal{O}(m)$	Suffix Tree
k-errors	$\mathcal{O}\left(m^k \sigma^k \max(k, \lg\lg n) + occ\right)$	[Lam et al., 2007]

Notations

Prefix and Suffix

$$P_i = T[1..i] \text{ is the } i\text{-th prefix of } T \text{ for all } i \in [1, n]$$
$$S_i = T[i..n] \text{ is the } i\text{-th suffix of } T \text{ for all } i \in [1, n]$$

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$T = \texttt{banana\$}$

i	1	2	3	4	5	6	7
S_i	banana$	anana$	nana$	ana$	na$	a$	$

The Suffix Array

Suffix Array of T

The SA is a permutation of $[1, n]$ such that for all $i \in [1, n-1]$:

$$T\left[SA\left[i\right]..n\right] <_{\text{lex}} T\left[SA\left[i+1\right]..n\right]$$

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$T =$ `banana$`

	1	2	3	4	5	6	7
$SA\left[i\right]$	7	6	4	2	1	5	3
	$	a	a	a	b	n	n
		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
					$		

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			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
					$		

Suffix Array Interval (SAI) of P

$$i \in \mathcal{I}(P) \iff T[SA[i]..SA[i]+|P|-1] = P$$

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$\mathcal{I}(\texttt{a}) = [2, 4]$

	1	2	3	4	5	6	7
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	$	a	a	a	b	n	n
		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
					$		

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$\mathcal{I}(\texttt{a}) = [2, 4]$

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	1	2	3	4	5	6	7
$SA\,[i]$	7	6	4	2	1	5	3
	\$	a	a	a	b	n	n
		\$	n	n	a	a	a
			a	a	n	\$	n
			\$	n	a		a
				a	n		\$
				\$	a		
					\$		

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	1	2	3	4	5	6	7
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	$	a	a	a	b	n	n
		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
					$		

Suffix Array Interval (SAI) of P

$$i \in \mathcal{I}(P) \iff T[SA[i]..SA[i] + |P| - 1] = P$$

The Inverse Suffix Array

Inverse Suffix Array of T

The SA^{-1} is a permutation of $[1, n]$ such that for all $i \in [1, n]$:

$$SA^{-1}[SA[i]] = i$$

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$\mathcal{I}(\texttt{a}) = [2, 4]$

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	1	2	3	4	5	6	7
$SA[i]$	7	6	4	2	1	5	3
$SA^{-1}[i]$	5	4	7	3	6	2	1
	$	a	a	a	b	n	n
		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
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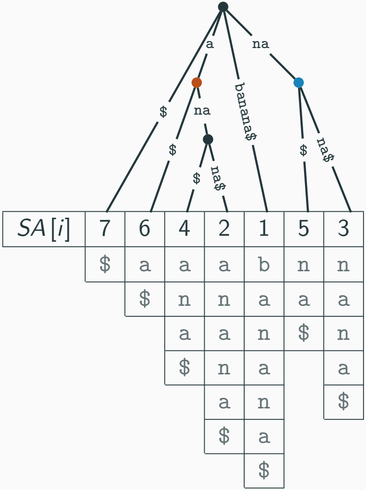
	1	2	3	4	5	6	7
$SA[i]$	7	6	4	2	1	5	3
$\Psi^1[i]$	-	1	6	7	4	2	3
	\$	a	a	a	b	n	n
		\$	n	n	a	a	a
			a	a	n	\$	n
			\$	n	a		a
				a	n		\$
				\$	a		
					\$		

Find the rest of the suffix

$$\Psi^k[i] = SA^{-1}[SA[i] + k]$$

The Suffix Tree



Tree above the Suffix Array

- Nodes cover *relevant SAIs*

$$\mathcal{I}(\mathtt{a}) = [2, 4]$$
$$\mathcal{I}(\mathtt{n}) = [6, 7]$$

The Idea

Find occurrences of subpatterns and merge suffix array intervals

Suffix Array Interval Merging

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Find occurrences of subpatterns and merge suffix array intervals

The Problem

How to find the interval gained by merging two suffix array intervals?

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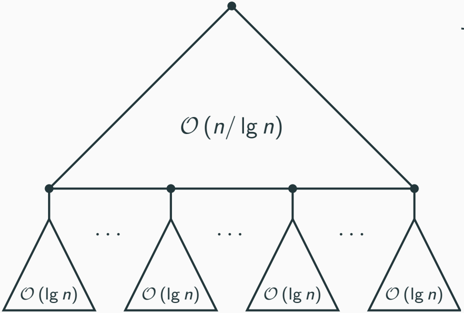
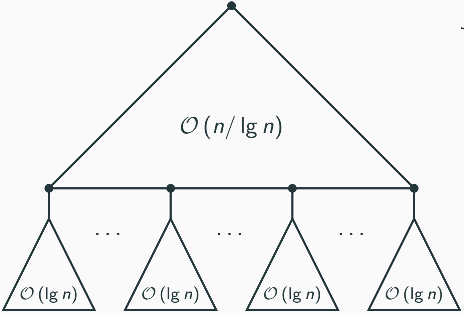
Paper	Running Time	Idea
[Huynh et al., 2006]	$\mathcal{O}(\lg n)$	Binary Search
[This talk]	$\mathcal{O}(\lg \lg n)$	Extending [Lam et al., 2007], Sampling Ψ in y-fast trie
	$\mathcal{O}\left(\lg_p \lg n\right)$	Parallel Binary Search



y-**Fast Trie** **[Willard, 1983]**

- Each leaf stores $\mathcal{O}(\lg n)$ elements in a binary search tree
- *x*-fast trie for $\mathcal{O}(n/\lg n)$ elements
- Prefixes of elements in $\mathcal{O}(\lg n)$ hash tables

Integer Dictionaries



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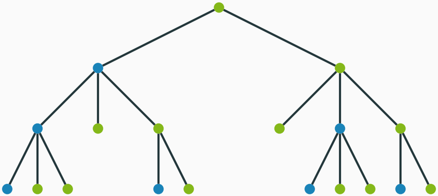
Find, Predecessor and Successor in $\mathcal{O}(\lg\lg n)\ldots$

- ... expected time **or**
- ... deterministic time with $\mathcal{O}(n \lg\lg n)$ construction time.

Heavy Path Decomposition

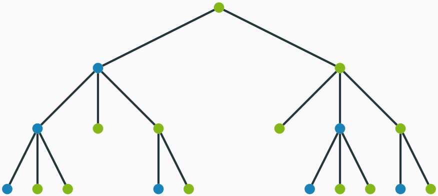


Nodes are

Heavy if they are in the largest subtree

Light otherwise (or if they are the root)

Heavy Path Decomposition



Nodes are

Heavy	if they are in the largest subtree
Light	otherwise (or if they are the root)

Sample Ψ for each light node

Sampling Ψ – Light nodes

Given two *SAIs* $\mathcal{I}(\alpha)$ **and** $\mathcal{I}(\beta)$

- Find all $i \in \mathcal{I}(\alpha) : \Psi^{|\alpha|}[i] \in \mathcal{I}(\beta)$
- $\Psi^{|\alpha|}[i]$ is monotonically increasing for all $i \in \mathcal{I}(\alpha)$

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Sampling for Light Nodes v **of** $\mathcal{I}(\alpha)$ **in** y**-fast trie**

$$\Gamma(v) := \left\{ \left(\Psi^{|\alpha|}[i], i\right) : i \equiv 1 \pmod{\lg^2 n} \wedge i \in \mathcal{I}(\alpha) \right\}$$

$\Psi^{|\alpha|}$			$\dots$				$\dots$				$\dots$			

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$$\Psi^{|\alpha|} \quad \square \quad \cdots \quad \square \quad \cdots \quad \square \quad \cdots \quad \square$$

Merging *SAI*s – Light nodes

Let v be the light node of $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta) = [b_\beta, e_\beta]$

- If $\Gamma(v) = \emptyset \rightarrow$ Binary search on $< \lg^2 n$ elements

$\psi^{|\alpha|}$

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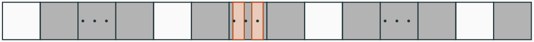

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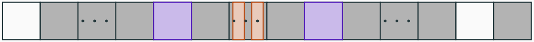
$\Psi^{|\alpha|}$



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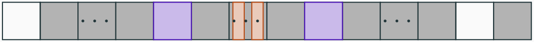
  Find k_l, k_r : $\Psi^{|\alpha|}[k_l] \leq b_\beta$ **and** $e_\beta \leq \Psi^{|\alpha|}[k_r]$
- Shrink k_l, k_r using binary search on $< \lg^2 n$ elements

$\Psi^{|\alpha|}$



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Similar idea for heavy nodes

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Similar idea for heavy nodes

Lemma

We can merge two SAIs in $\mathcal{O}(\lg \lg n)$ time.

Parallelize the Merging

What are we doing to merge two *SAI*s

Query y-fast tries **and** binary search

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Parallelize these queries

- Binary search requires $\mathcal{O}\left(\lg_p n\right)$ parallel time [Snir 1985]
- Binary search in the x-fast trie
- Static y-fast trie $\rightarrow$ arrays instead of binary search trees

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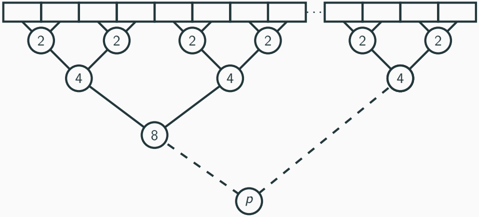
Lemma

We can merge two SAIs in $\mathcal{O}\left(\lg_p \lg n\right)$ parallel time.

Parallel Exact Pattern Matching

- $P = P_1 P_2 \ldots P_p$ with $|P_i| = m/p$
- Compute $\mathcal{I}(P_i)$ in $\mathcal{O}\left(m/p\right)$ time
- Merge *SAI*s in $\mathcal{O}\left(\lg_p \lg n\right)$ time



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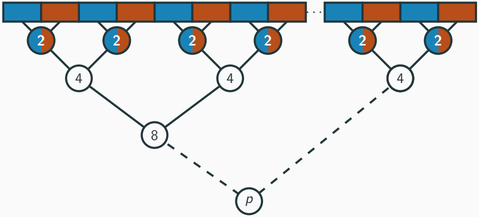

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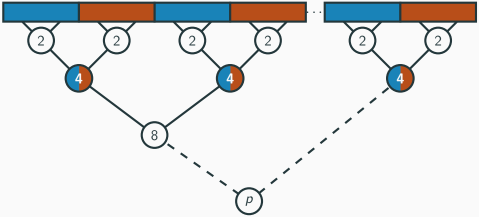

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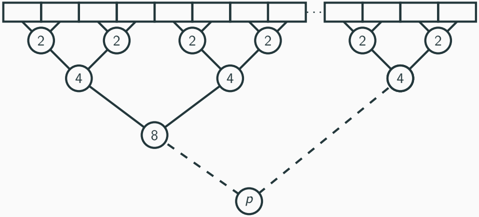

In the k-th Step

$p/2^k$ *SAI*s $\rightarrow 2^k$ processors

Number of Steps

There are $\lg p$ merge steps

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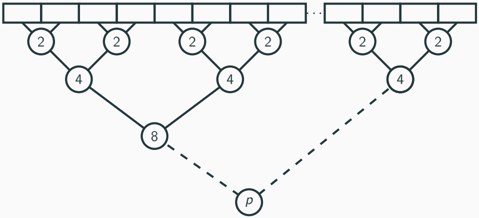

In the k-th Step

$p/2^k$ *SAI*s $\rightarrow 2^k$ processors

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Theorem

Parallel exact pattern matching requires $\mathcal{O}(m/p + \lg\lg p \lg\lg n)$ *time.*

The k-Difference and k-Mismatch Problem

Given a text T of length n and a pattern P of length m . . .

k-Difference Problem

. . . find all occurrences of P' in T such that P can be transformed to P' using $\leq k$ Insert, Change and Delete operations.

The k-Difference and k-Mismatch Problem

Given a text T of length n and a pattern P of length m ...

k-Difference Problem

... find all occurrences of P' in T such that P can be transformed to P' using $\leq k$ INSERT, CHANGE and DELETE operations.

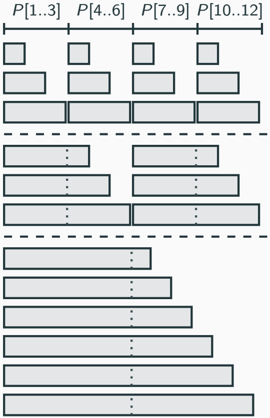
k-Mismatch Problem

... find all occurrences of P' in T such that P can be transformed to P' using $\leq k$ CHANGE operations.

Preprocessing

Compute *SAI*s of all prefixes and suffixes of P

Preprocessing: $|P| = 12, p = 4$



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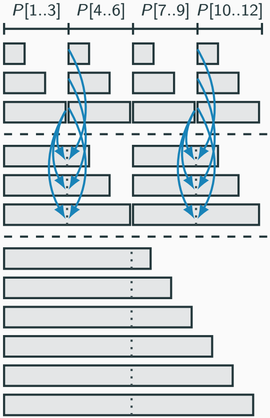

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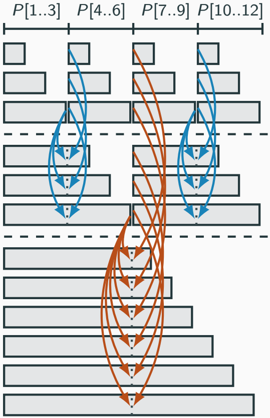

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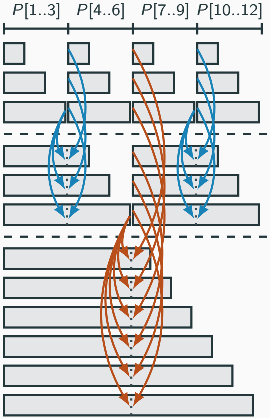

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- There are $\lg n$ merge steps
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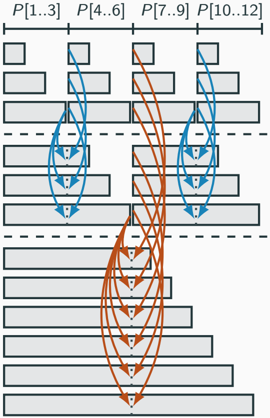

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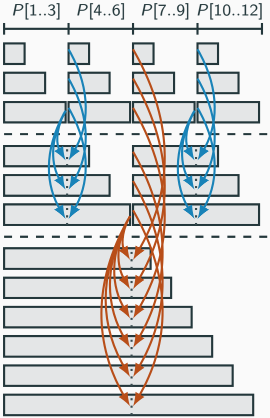

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Lemma

The preprocessing requires $\mathcal{O}\left(m/p \lg p \lg \lg n\right)$ time.

Introducing the Error (Insert, Change or Delete)

- $\mathcal{I}(P\,[1..i])$ and $\mathcal{I}(P\,[i..n])$ are known
- What is an error at position j

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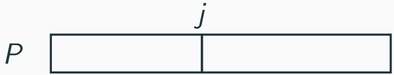

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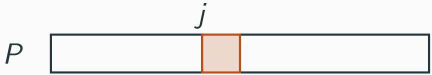

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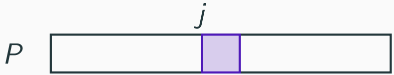

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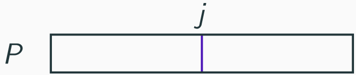

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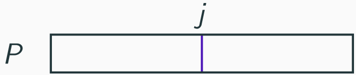

Theorem

Approximate parallel pattern matching with ≤ 1 error can be solved in $\mathcal{O}(\sigma m/p \cdot \lg\lg n + occ)$ time.

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Quite similar to $k = 1$

- The same preprocessing
- Introduce $\leq k$ errors by merging SAIs
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Problem – Report Occurrence Multiple Times

The Problem: $T = \texttt{aaa\$}$ **and** $P = \texttt{aba}$ **and one error**

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Both P' and P'' occur at position 1 in T

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The Solution

- Report only if found with smallest distance [Huynh et al., 2006]
- Can be parallelized

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Things we did

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Thank You