On the Benefit of Merging Suffix Array Intervals for Parallel Pattern Matching

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71. Workshop über Algorithmen und Komplexität

Notations

- Σ is the alphabet with $|\Sigma| = \sigma$
- $\$ \notin \Sigma$ and $\forall \alpha \in \Sigma : \$ <_{\text{lex}} \alpha$
- $T \in \Sigma^\star \cup \{\$\}$ and $P \in \Sigma^\star$
- $|T| = n$ and $|P| = m$
- p is the number of processors

Pattern Matching

Given a text T of length n and a pattern P of length m, find all occurrences of P in T.

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Sequential Times

Type	Query Time	Idea
exact	$\mathcal{O}(m)$	Suffix Tree
k-errors	$\mathcal{O}\left(m^k \sigma^k \max(k, \lg\lg n) + occ\right)$	[Lam et al., 2007]

Notations

Prefix and Suffix

$$P_i = T[1..i] \text{ is the } i\text{-th prefix of } T \text{ for all } i \in [1, n]$$
$$S_i = T[i..n] \text{ is the } i\text{-th suffix of } T \text{ for all } i \in [1, n]$$

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i	1	2	3	4	5	6	7
S_i	banana$	anana$	nana$	ana$	na$	a$	$

The Suffix Array

Suffix Array of T

The SA is a permutation of $[1, n]$ such that for all $i \in [1, n-1]$:

$$T[SA[i]..n] <_{\text{lex}} T[SA[i+1]..n]$$

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$T =$ `banana$`

	1	2	3	4	5	6	7
$SA[i]$	7	6	4	2	1	5	3
	$	a	a	a	b	n	n
		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
					$		

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			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
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Suffix Array Interval (SAI) of P

$$i \in \mathcal{I}(P) \iff T[SA[i]..SA[i] + |P| - 1] = P$$

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$\mathcal{I}(\texttt{a}) = [2, 4]$

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		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
					$		

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$SA[i]$	7	6	4	2	1	5	3
	\$	a	a	a	b	n	n
		\$	n	n	a	a	a
			a	a	n	\$	n
			\$	n	a		a
				a	n		\$
				\$	a		
					\$		

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The Inverse Suffix Array

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The SA^{-1} is a permutation of $[1, n]$ such that for all $i \in [1, n]$:

$$SA^{-1}[SA[i]] = i$$

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	1	2	3	4	5	6	7
$SA[i]$	7	6	4	2	1	5	3
$SA^{-1}[i]$	5	4	7	3	6	2	1
	$	a	a	a	b	n	n
		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
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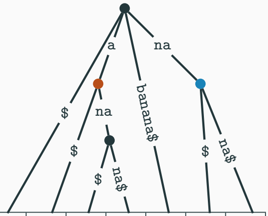
	1	2	3	4	5	6	7
$SA[i]$	7	6	4	2	1	5	3
$\Psi^1[i]$	-	1	6	7	4	2	3
	$	a	a	a	b	n	n
		$	n	n	a	a	a
			a	a	n	$	n
			$	n	a		a
				a	n		$
				$	a		
					$		

Find the rest of the suffix

$$\Psi^k[i] = SA^{-1}[SA[i] + k]$$

The Suffix Tree



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	$	a	a	a	b	n	n
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					$		

Tree above the Suffix Array

- Nodes cover *relevant SAIs*

$$\mathcal{I}(\texttt{a}) = [2,4]$$
$$\mathcal{I}(\texttt{n}) = [6,7]$$

The Idea

Find occurrences of subpatterns and merge suffix array intervals

Suffix Array Interval Merging

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The Problem

How to find the interval gained by merging two suffix array intervals?

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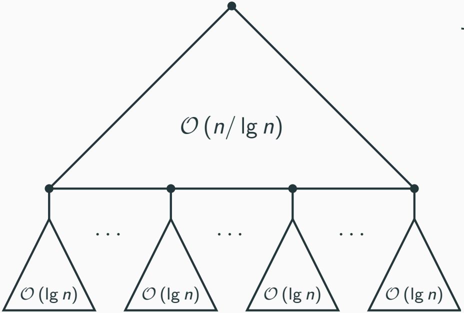
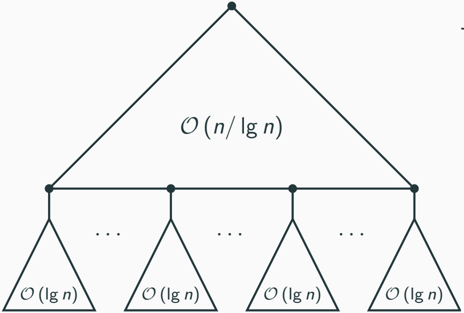
Paper	Running Time	Idea
[Huynh et al., 2006]	$\mathcal{O}(\lg n)$	Binary Search
[This talk]	$\mathcal{O}(\lg \lg n)$	Extending [Lam et al., 2007], Sampling Ψ in y-fast trie
	$\mathcal{O}\left(\lg_p \lg n\right)$	Parallel Binary Search



y-**Fast Trie** **[Willard, 1983]**

- Each leaf stores $\mathcal{O}(\lg n)$ elements in a binary search tree
- *x*-fast trie for $\mathcal{O}(n/\lg n)$ elements
- Prefixes of elements in $\mathcal{O}(\lg n)$ hash tables

Integer Dictionaries



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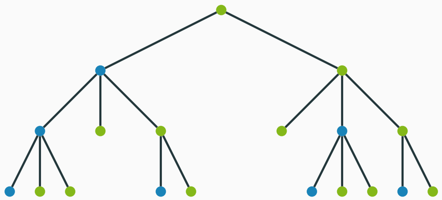
FIND, PREDECESSOR and SUCCESSOR in $\mathcal{O}(\lg \lg n)\ldots$

- ... expected time **or**
- ... deterministic time with $\mathcal{O}(n \lg \lg n)$ construction time.

Heavy Path Decomposition



Nodes are

Heavy if they are in the largest subtree

Light otherwise (or if they are the root)

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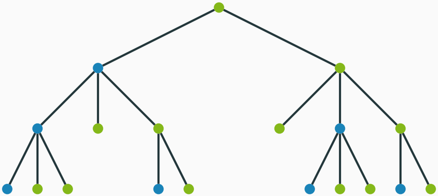

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Sample Ψ for each light node

Sampling Ψ – Light nodes

Given two *SAIs* $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta)$

- Find all $i \in \mathcal{I}(\alpha) : \Psi^{|\alpha|}[i] \in \mathcal{I}(\beta)$
- $\Psi^{|\alpha|}[i]$ is monotonically increasing for all $i \in \mathcal{I}(\alpha)$

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Sampling for Light Nodes v **of** $\mathcal{I}(\alpha)$ **in** y**-fast trie**

$$\Gamma(v) := \left\{ \left(\Psi^{|\alpha|}[i], i\right) : i \equiv 1 \pmod{\lg^2 n} \wedge i \in \mathcal{I}(\alpha) \right\}$$

$\Psi^{\lvert\alpha\rvert}$			$\dots$				$\dots$				$\dots$			

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$\Psi^{|\alpha|}$ $\square$ $\cdots$ $\square$ $\cdots$ $\square$ $\cdots$ $\square$

Merging *SAI*s – Light nodes

Let v be the light node of $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta) = [b_\beta, e_\beta]$

- If $\Gamma(v) = \emptyset \rightarrow$ Binary search on $< \lg^2 n$ elements

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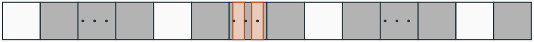

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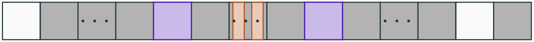

Merging *SA*Is – Light nodes

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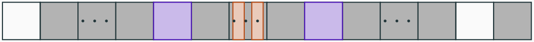
  Find k_l, k_r : $\Psi^{|\alpha|}[k_l] \leq b_\beta$ **and** $e_\beta \leq \Psi^{|\alpha|}[k_r]$

- Shrink k_l, k_r using binary search on $< \lg^2 n$ elements

$\Psi^{|\alpha|}$



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Similar idea for heavy nodes

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Lemma

We can merge two SAIs in $\mathcal{O}(\lg \lg n)$ time.

Parallelize the Merging

What are we doing to merge two *SAI*s

Query y-fast tries **and** binary search

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Parallelize these queries

- Binary search requires $\mathcal{O}\left(\lg_p n\right)$ parallel time [Snir 1985]
- Binary search in the x-fast trie
- Static y-fast trie $\rightarrow$ arrays instead of binary search trees

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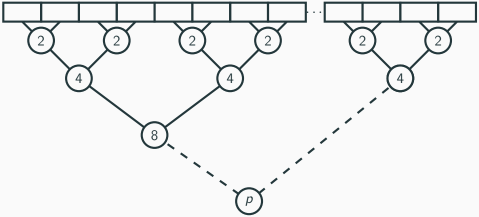
Lemma

We can merge two SAIs in $\mathcal{O}\left(\lg_p \lg n\right)$ *parallel time.*

Parallel Exact Pattern Matching

- $P = P_1 P_2 \ldots P_p$ with $|P_i| = m/p$
- Compute $\mathcal{I}(P_i)$ in $\mathcal{O}(m/p)$ time
- Merge *SAI*s in $\mathcal{O}\left(\lg_p \lg n\right)$ time



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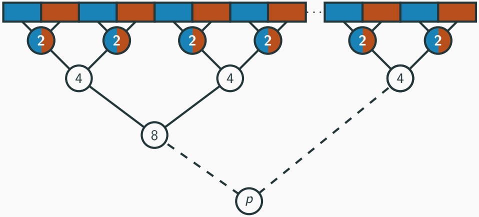

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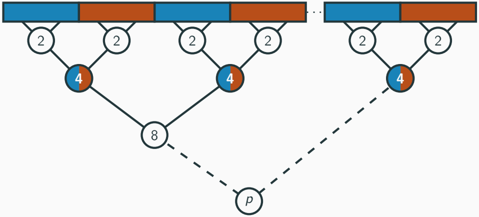

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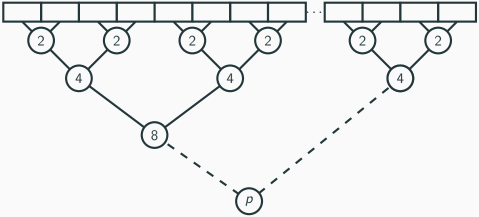

In the k-th Step

$p/2^k$ *SAI*s $\rightarrow 2^k$ processors

Number of Steps

There are $\lg p$ merge steps

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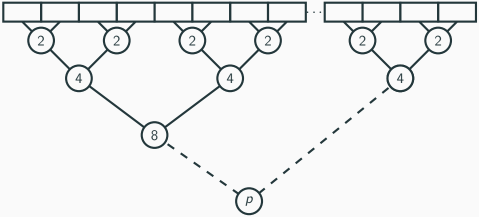

In the k-th Step

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Number of Steps

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Theorem

Parallel exact pattern matching requires $\mathcal{O}\left(m/p + \lg\lg p \lg\lg n\right)$ *time.*

The k-Difference and k-Mismatch Problem

Given a text T of length n and a pattern P of length m ...

k-Difference Problem

... find all occurrences of P' in T such that P can be transformed to P' using $\leq k$ Insert, Change and Delete operations.

The k-Difference and k-Mismatch Problem

Given a text T of length n and a pattern P of length m . . .

k-Difference Problem

. . . find all occurrences of P' in T such that P can be transformed to P' using $\leq k$ INSERT, CHANGE and DELETE operations.

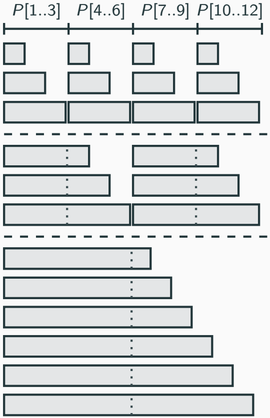
k-Mismatch Problem

. . . find all occurrences of P' in T such that P can be transformed to P' using $\leq k$ CHANGE operations.

Preprocessing

Compute *SAI*s of all prefixes and suffixes of P

Preprocessing: $|P| = 12, p = 4$



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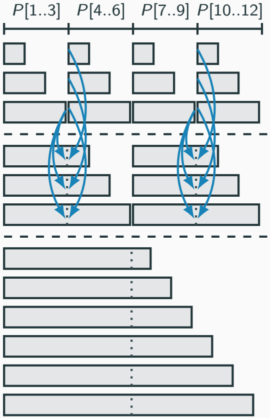

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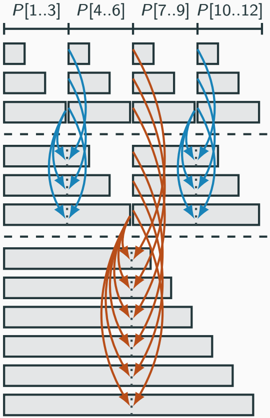

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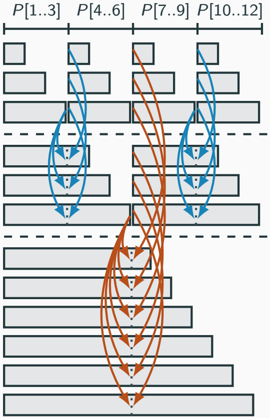

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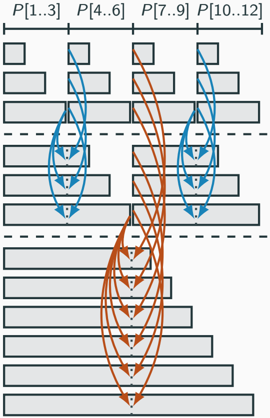

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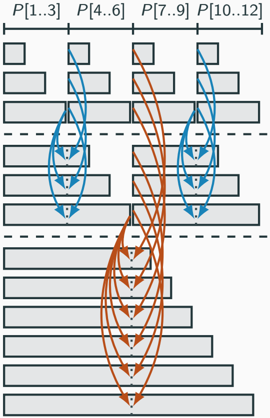

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Lemma

The preprocessing requires $\mathcal{O}\left(m/p \lg p \lg \lg n\right)$ time.

Solving the 1-Difference and 1-Mismatch Problem

Introducing the Error (Insert, Change or Delete)

- $\mathcal{I}(P\,[1..i])$ and $\mathcal{I}(P\,[i..n])$ are known
- What is an error at position j

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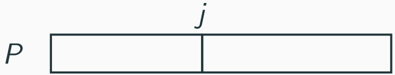

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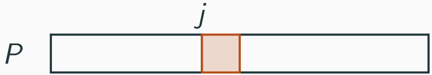

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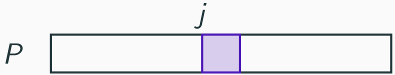

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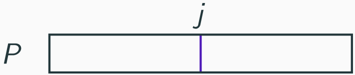

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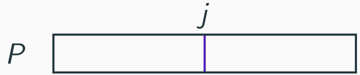

Theorem

Approximate parallel pattern matching with ≤ 1 error can be solved in $\mathcal{O}(\sigma m/p \cdot \lg\lg n + occ)$ time.

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Quite similar to $k = 1$

- The same preprocessing
- Introduce $\leq k$ errors by merging SAIs
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Problem – Report Occurrence Multiple Times

The Problem: $T = \texttt{aaa\$}$ **and** $P = \texttt{aba}$ **and one error**

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Both P' and P'' occur at position 1 in T

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The Solution

- Report only if found with smallest distance [Huynh et al., 2006]
- Can be parallelized

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Things we did

- Presented efficient parallel algorithm for merging SAIs
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Thank You