Advanced Data Structures

Lecture 06: BSP Trees and Packed and Compressed Hash Tables

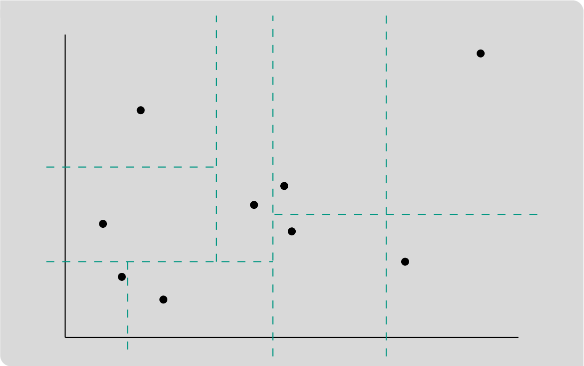
Florian Kurpicz

Recap: 2-Dimensional Rectangular Range Searching

Important

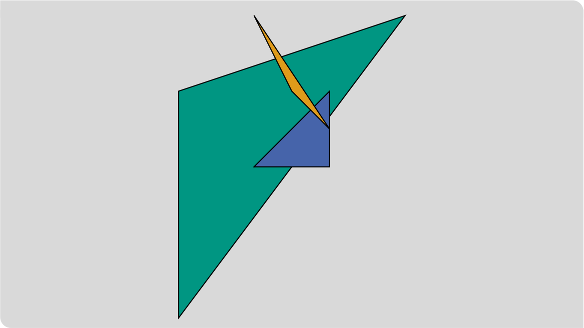
- assume now two points have the same x- or y-coordinate

- generalize 1-dimensional idea
- 1-dimensional
  - split number of points in half at each node
  - points consist of one value
- 2-dimensional
  - points consist of two values
  - split number of points in half w.r.t. one value
  - switch between values depending on depth



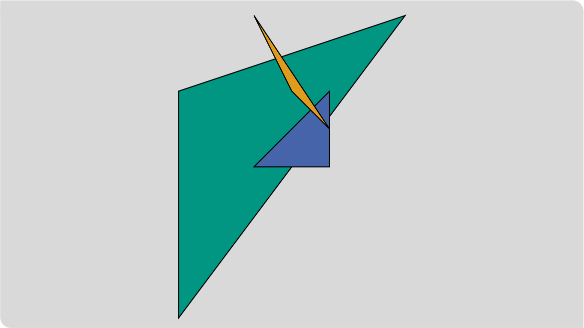
Motivation

- hidden surface removal
- which pixel is visible
- important for rendering



z-Buffer Algorithm

- transform scene such that viewing direction is positive z-direction
- consider objects in scene in arbitrary order
- maintain two buffers
  - frame buffer ℹ currently shown pixel
  - z-buffer ℹ z-coordinate of object shown
- compare z-coordinate of z-buffer and object



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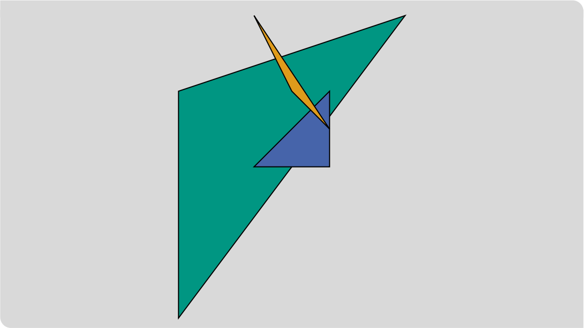
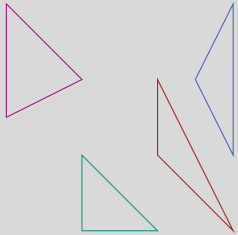


- first sort object in depth-order
- depth-order may not always exist
- how to efficiently sort objects?

BSP Trees (1/2)

- partition space using hyperplanes
- binary partition ℹ similar to kd-tree
- hyperplanes create half-spaces and cut objects into fragments



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- $h^+ = \{(x_1, \ldots, x_d) \colon a_1 x_1 + \cdots + a_d x_d > 0\}$
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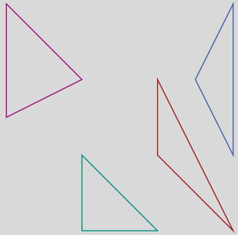

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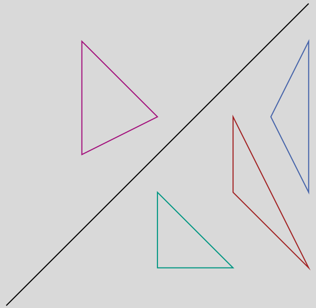

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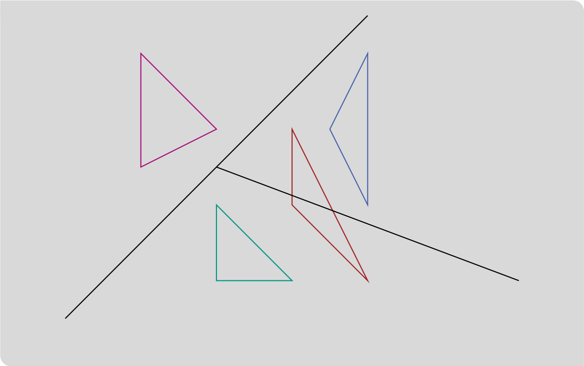

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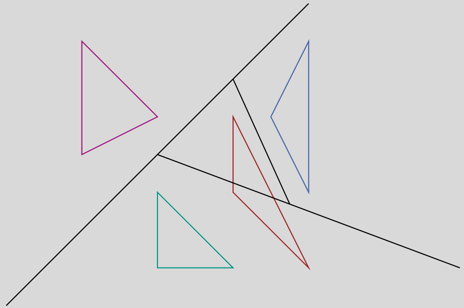

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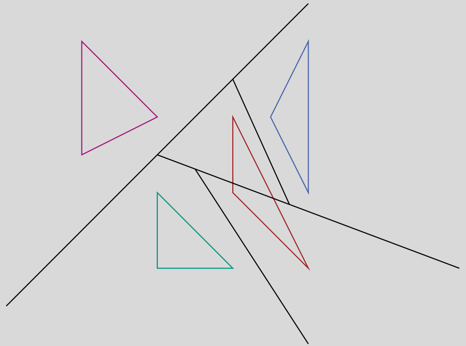

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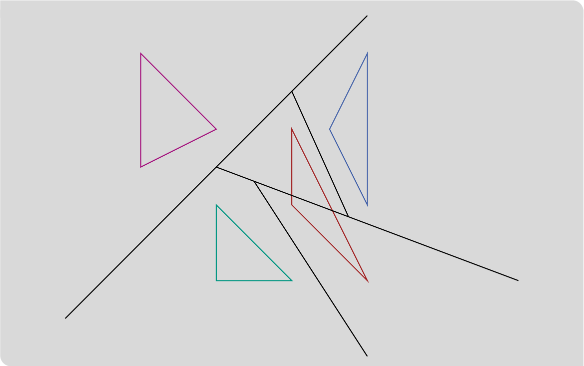
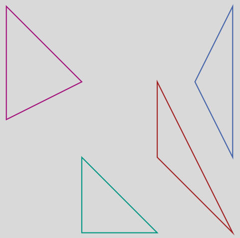
- each split creates two nodes in a tree
- if number of objects in space is one: leaf
- otherwise: inner node



BSP Trees (2/2)

- for leaf: store object/fragment
- for inner node v: store hyperplane h_v and the objects contained in h_v
- left child represents objects in upper half-space h^+
- right child represents objects in lower half-space h^-



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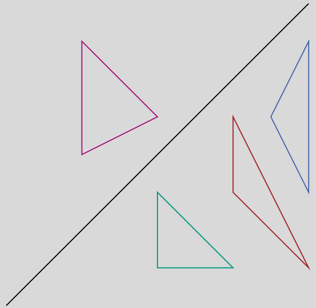

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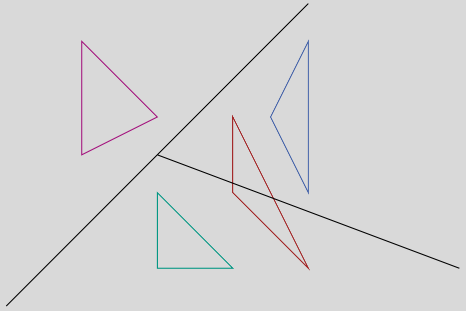

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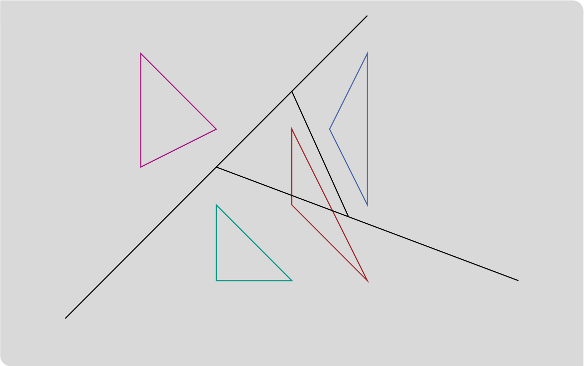

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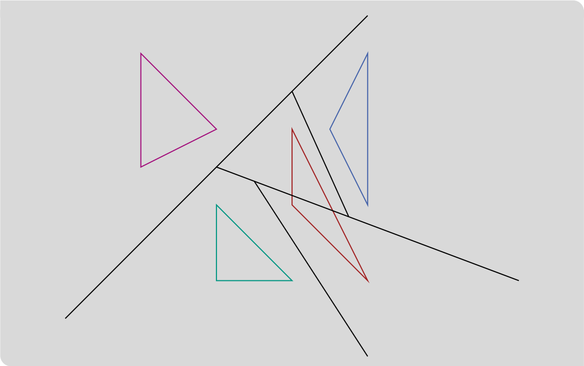

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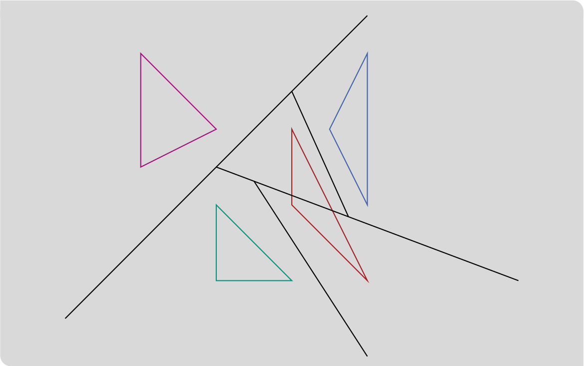
- space of BSP tree is number of objects stored at all nodes
- what about fragments?
- too many fragments can make the tree big



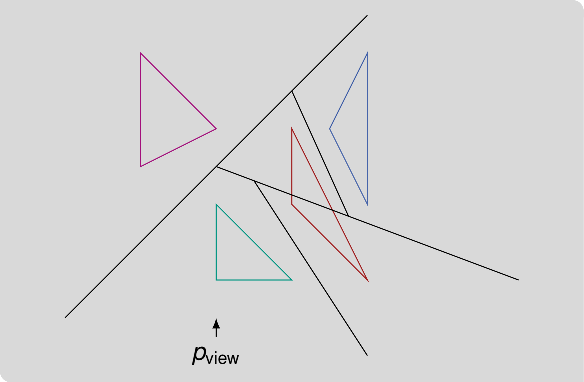
Auto-Partitioning

Karlsruhe Institute of Technology

- sorting points for kd-trees worked well
- BSP-tree is used to sort objects in dept-order
- auto-partitioning uses splitters through objects
  - 2-dimensional: line through line segments
  - 3-dimensional: half-plane through polygons

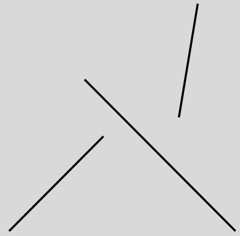
Painter's Algorithm

- consider view point p_{view}
- traverse through tree and always recurse on half-space that does not contain p_{view} first
- then scan-convert object contained in node
- then recurse on half-space that contains p_{view}



Constructing Planar BSP Trees (1/3)

- use auto-partitioning
- construction similar to construction of kd-tree
- store all necessary information
  - hyperplane
  - objects in hyperplane
- how to determine next hyperplane?
- creating fragments increases size of BSP tree



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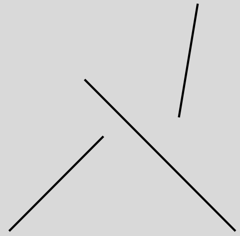
- let s be object and $\ell(s)$ line through object
- order matters



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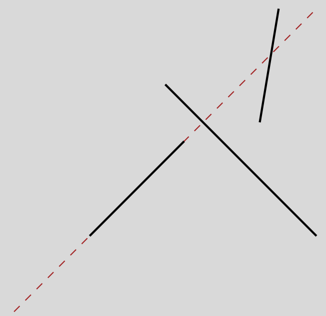

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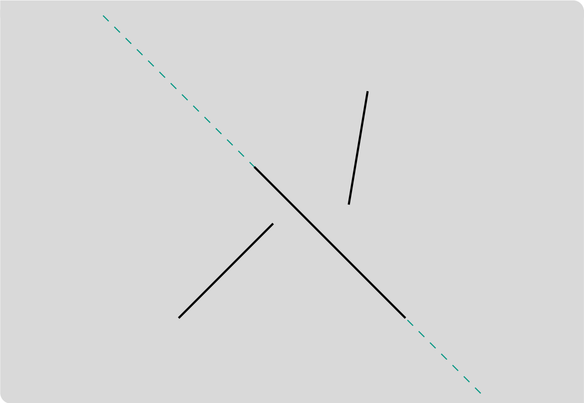

Constructing Planar BSP Trees (2/3)

Lemma: Number Line Fragments

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Proof (Sketch)

- distance of lines $dist_{s_i}(s_j) =$
$$\begin{cases} \text{\# segments inters. } \ell(s_i) & \\ \text{ between } s_i \text{ and } s_j & \ell(s_i) \text{ inters. } s_j \\ \infty & \text{otherwise} \end{cases}$$
- example on the board

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Proof (Sketch, cnt.)

- let $dist_{s_i}(s_j) = k$ and $s_{j_1}, \dots, s_{j_k}$ be segments between s_i and s_j
- what is the probability that $\ell(s_i)$ cuts s_j?
- this happens if no s_{j_x} is processed before s_i
- since order is random

$$\mathbb{P}[\ell(s_i) \text{ cuts } s_j] \leq \frac{1}{dist_{s_i}(s_j) + 2}$$

Constructing Planar BSP Trees (3/3)

Proof (Sketch, cnt.)

- expected number of cuts

$$\mathbb{E}[\text{\# cuts generated by } s_i] \leq \sum_{j \neq i} \frac{1}{dist_{s_i}(s_j) + 2} \leq 2 \sum_{k=0}^{n-2} \frac{1}{k+2} \leq 2 \ln n$$

- all lines generate at most $2n \ln n$ fragments

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Lemma: BSP Construction

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Proof (Sketch)

- computing permutation in linear time
- construction is linear in number of fragments to be considered
- number of fragments in subtree is bounded by n
- number of recursions is $n \log n$

New Topic: Hash Tables

Karlsruhe Institute of Technology

- now hash tables
- first packed and compressed hash table
- presented in January ’23 at ALENEX

Motivation

Setting

- static hash table for objects of variable size
- storing objects in external memory
- ideally retrieve objects in single I/O
- very small internal memory data structure

Objects of Variable Size

External Memory

- only blocks of size B bits can be transferred
- one I/O per block transfer

Space-Efficient Object Stores from Literature

- objects of size 256 bytes
- blocks of size 4096 bytes
- internal space I_b (bits/block)
- (*) consecutive I/O

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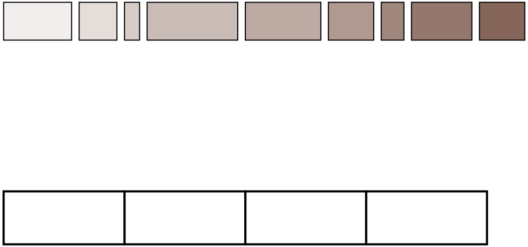
	Method	I_b	load factor	I/Os
fixed	Larson et al. [LR85]	96	$<$96 %	1
	SILT SortedStore [Lim+11]	51	100 %	1
	Linear Separator [Lar88]	8	85 %	1
	Separator [GL88; LK84]	6	98 %	1
	Robin Hood [Cel88]	3	99 %	1.3
	Ramakrishna et al. [RT89]	4	80 %	1
	Jensen, Pagh [JP08]	0	80 %	1.25
	Cuckoo [Aza+94; Pag03]	0	$<$100 %	2
	PaCHash, $a = 1$	2	100 %	2*
	PaCHash, $a = 8$	5	100 %	1.13*
variable	SILT LogStore [Lim+11]	832	100 %	1
	SkimpyStash [DSL11]	32	$\leq$98 %	8
	PaCHash, $a = 1$	2	99.95 %	2.06*
	PaCHash, $a = 8$	5	99.95 %	1.19*

PaCHash Overview



objects of variable size

EM

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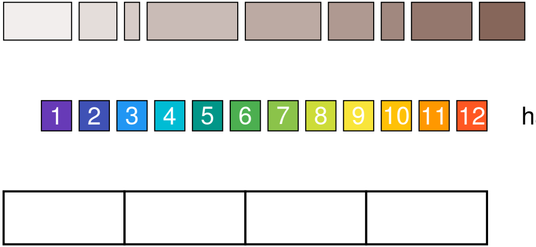

objects of variable size

hash function $h \colon K \to 1..am$

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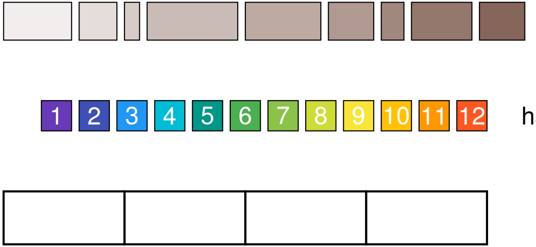

objects of variable size

tuning parameter

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EM blocks

EM

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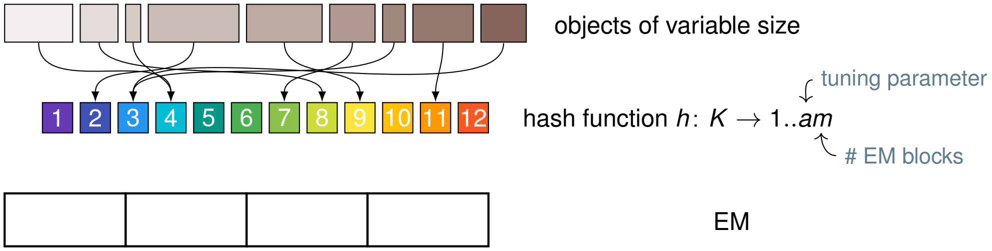

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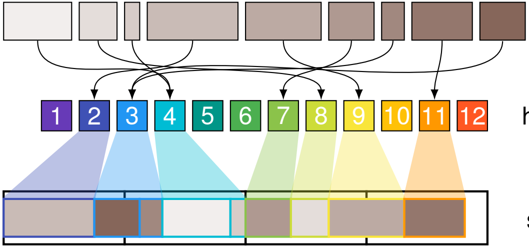

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EM blocks

sorted objects in EM in bins

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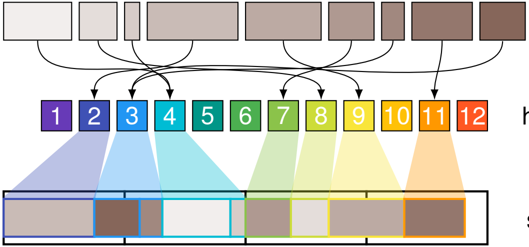

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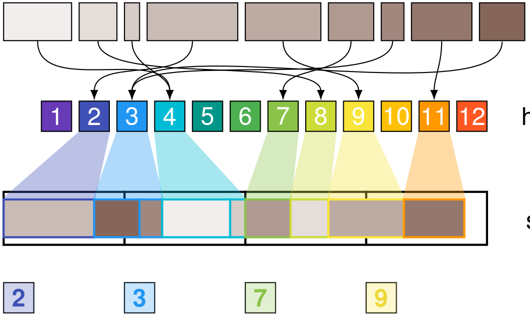

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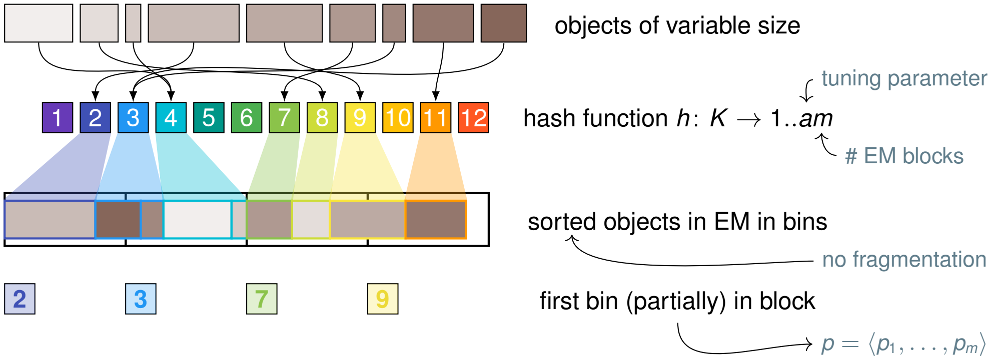

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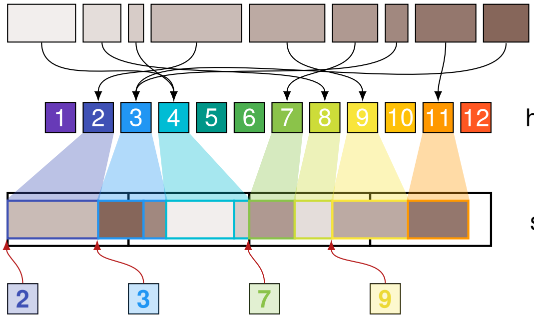

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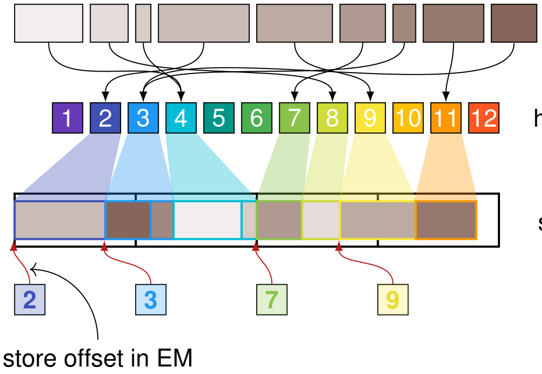

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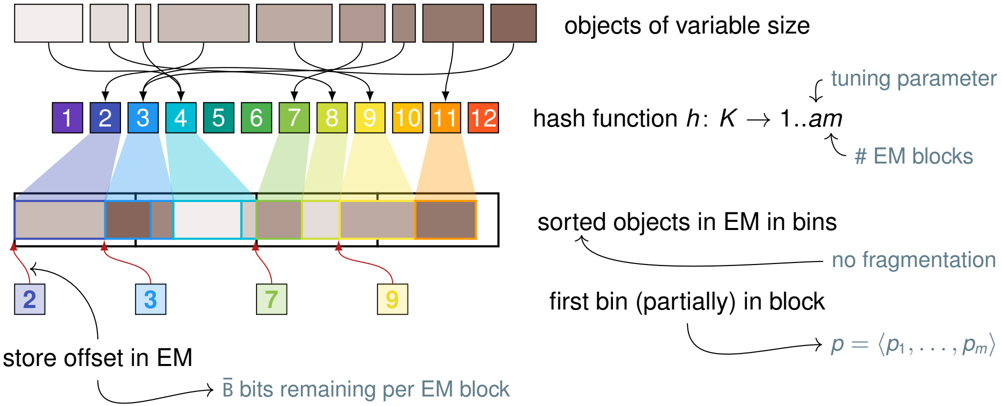
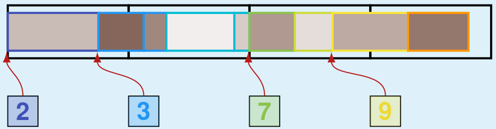


Finding Blocks

Query Algorithm



- $b_x = h(x)$
- find first i with $p_i \leq b_x$
- if $p_i = b_x$ let $i = i - 1$
- find first j with $p_j > b_x$
- return $i..(j-1)$

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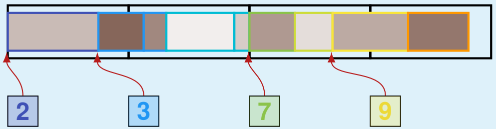

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Elias-Fano Coding

- given k monotonic increasing integers in $1..u$
  - store $\log k$ MSBs encoded in bit vector
  - store $\log(u/k)$ LSBs plain
  - $k(2 + \log(u/k)) + 1 + o(k)$ bits in total
- predecessor in $O(k)$ time

Finding Blocks

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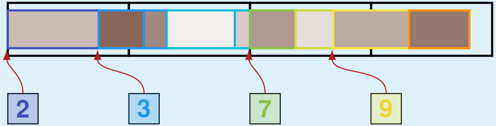

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- find first j with $p_j > b_x$
- return $i..(j-1)$

Elias-Fano Coding

- given k monotonic increasing integers in $1..u$
  - store $\log k$ MSBs encoded in bit vector
  - store $\log(u/k)$ LSBs plain
  - $k(2 + \log(u/k)) + 1 + o(k)$ bits in total
- predecessor in $O(k)$ time

Lemma: Space with Elias-Fano Coding

When using Elias-Fano coding [Eli74; Fan71] to store p, the index needs $2 + \log a + o(1)$ bits of internal memory per block.

Finding Blocks

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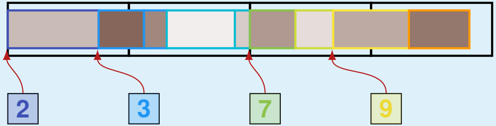
Query Algorithm



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Lemma: Expected Predecessor Time

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Proof (Sketch)

- consider $\lceil \log m \rceil$ MSB
- let bin b_x have MSBs equal to u
- expected size $\mathbb{E}(Y_u)$ of all bins with MSB u that are $< b_x$ is

$$\begin{aligned}
&\sum_{y \in S} |y| \cdot \mathbb{P}(h(y) \text{ w/ MSB} = u; h(y) < h(x)) \\
\leq &\sum_{y \in S} |y| \cdot \mathbb{P}(h(y) \text{ w/ MSB} = u) \\
= &\frac{1}{m} \sum_{y \in S} |y| = \frac{m\bar{\mathrm{B}}}{m} = \bar{\mathrm{B}}
\end{aligned}$$

- number of entries to scan is $\mathbb{E}(Y_u)/\bar{\mathrm{B}} = 1$

Loading Blocks from External Memory

Lemma: Additional Blocks Loaded

Retrieving an object x of size $|x|$ from a PaCHash data structure loads $\leq 1 + |x|/\overline{B} + 1/a$ consecutive blocks from the external memory in expectation.

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- expected size of bin $b_x = h(x)$

$$\mathbb{E}(|b_x|) = |x| + \sum_{y \in S, y \neq x} |y| \mathbb{P}(y \in b_x)$$

$$\leq |x| + \sum_{y \in S} |y| \mathbb{P}(y \in b_x)$$

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Proof (Sketch, cnt.)

- expected number of blocks overlapped by b_x

$$\mathbb{E}(X) = 1 + (\mathbb{E}(|b_x|) - 1)/\overline{B}$$
$$= 1 + \frac{|x|}{\overline{B}} + \frac{1}{a} - 1/\overline{B}$$

- $\mathbb{P}(\text{bin and block border align}) = 1/\overline{B}$

Experimental Evaluation

Hardware and Software

- Intel i7 11700 (base clock speed: 2.5 GHz)
- 1 TB Samsung 980 Pro NVMe SSD
- Ubuntu 21.10 (Kernel 5.13.0)
- `io_uring` for I/O operations
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- more in the paper (very similar results)

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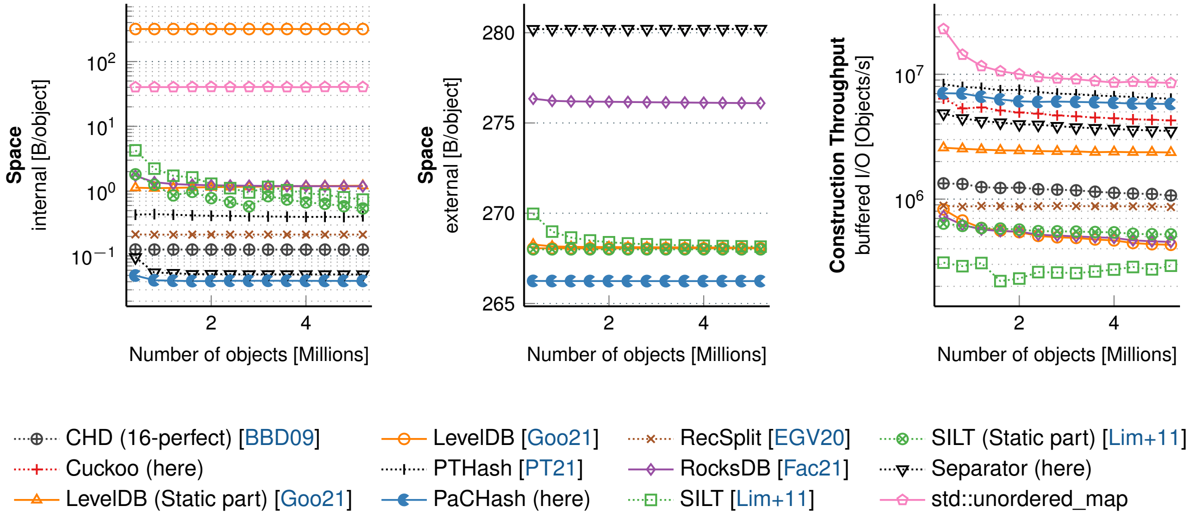
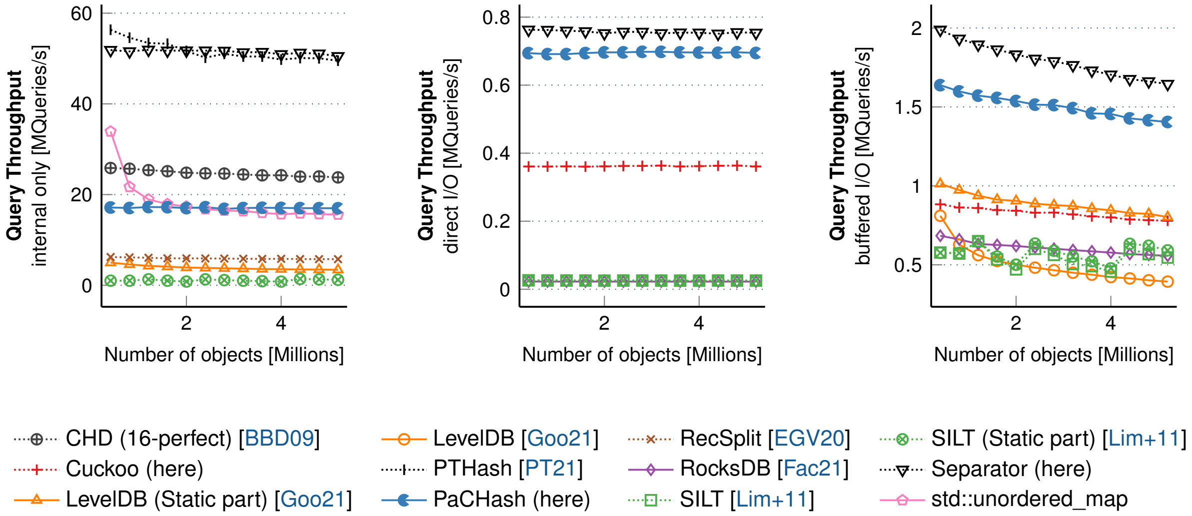
Competitors

- LevelDB [Goo21]
- RocksDB [Fac21]
- SILT [Lim+11].
- `std::unordered_map`
- RecSplit [EGV20]
- CHD [BBD09; CR+12]
- PTHash [PT21]

Construction

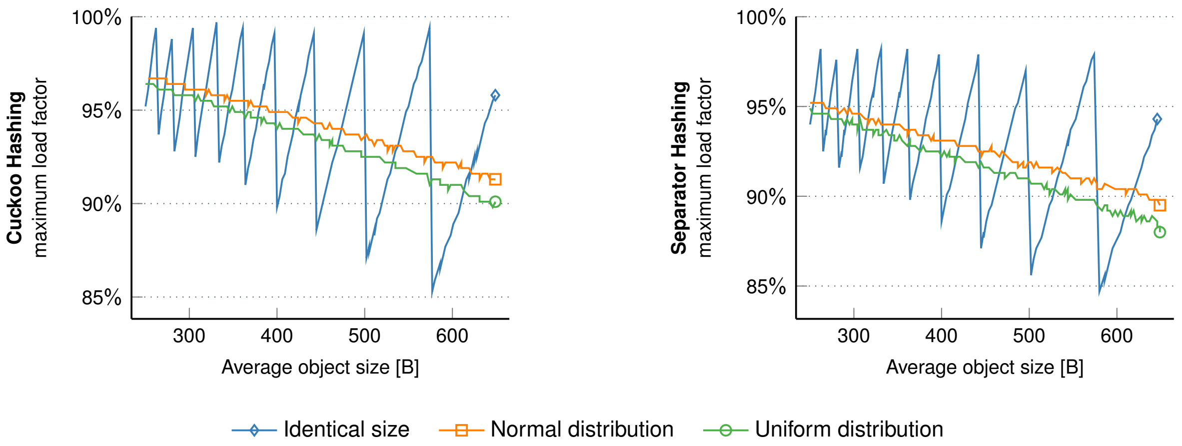


Queries



Query Throughput internal only [MQueries/s]
60
40
20
0
2
4
Number of objects [Millions]
Query Throughput direct I/O [MQueries/s]
0.8
0.6
0.4
0.2
0
2
4
Number of objects [Millions]
Query Throughput buffered I/O [MQueries/s]
2
1.5
1
0.5
2
4
Number of objects [Millions]
CHD (16-perfect) [BBD09]
Cuckoo (here)
LevelDB (Static part) [Goo21]
LevelDB [Goo21]
PTHash [PT21]
PaCHash (here)
RecSplit [EGV20]
RocksDB [Fac21]
SILT [Lim+11]
SILT (Static part) [Lim+11]
Separator (here)
std::unordered_map

Maximum Load Factor of Competitors



Alternative Internal Memory Data Structures

Lemma: Space with Succincter

When using Succincter [Pat08] to store p, the index needs $1.44 + \log(a+1) + o(1)$ bits of internal memory per block.

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KIT Karlsruhe Institute of Technology

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- runs of `0`s and `10`s
- sometimes additional `1`s

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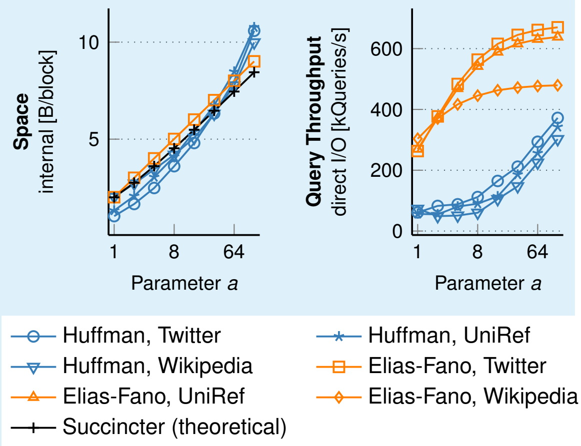

Conclusion and Outlook

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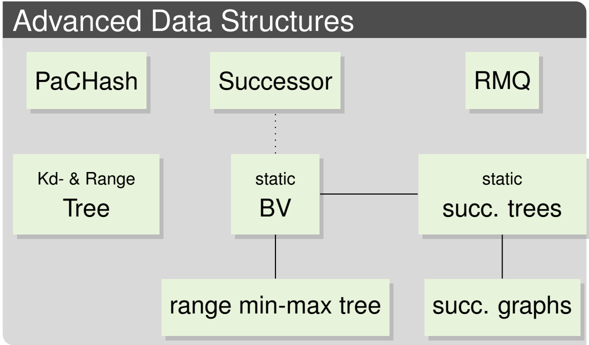
This Lecture

- BSP trees
- PaCHash



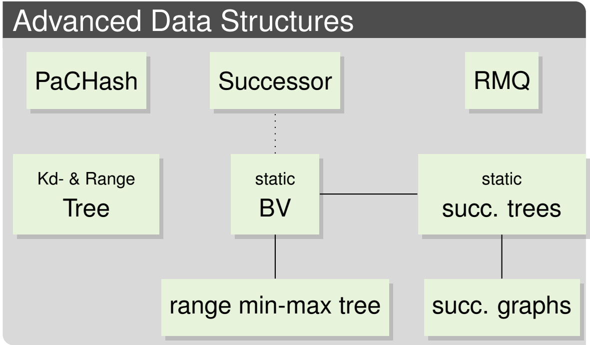
Conclusion and Outlook

This Lecture

- BSP trees
- PaCHash

Next Lecture

- more on hashing



F.A.Q. Project

- measuring memory

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- measuring memory
- measuring time

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- `std::map` vs `st::unordered_map`

F.A.Q. Project

- measuring memory
- measuring time
- `std::map` vs `st::unordered_map`
- more questions?

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